Lambda Abstraction as a Factor in Human Uniqueness

İsa Kerem Bayırlı

[1] Department of Turkish Language and Literature, TOBB University of Economics and Technology, Ankara, Turkey.

Abstract

There appear to be some qualitative differences between the conceptual repertoire of humans and that of other animals. We propose that the mental operation of Lambda Abstraction may shed some light on this distinction. More specifically, we claim that humans and only humans make use of mental representations constructed with the rule of Lambda Abstraction, which enables them to entertain concepts that can be used for entities that are not necessarily within their domain of experience. In addition to defining new types of concepts, Lambda Abstraction has played a crucial role in unlocking the potential for semantically consequential Internal Merge and quantification. This paper highlights the fact that research on language evolution should focus more on the transformative cognitive consequences of the interface between syntax and thought systems.

Keywords

lambda abstraction, concepts, merge, quantification, language evolution, human uniqueness

1 Introduction

There appear to be some qualitative differences between the conceptual repertoire of humans and that of other animals. We propose that the mental operation of Lambda Abstraction may shed some light on this distinction. More specifically, we claim that humans make use of mental representations constructed with the rule of Lambda Abstraction, which enables them to entertain concepts that can be used for entities that are not necessarily within their domain of experience (Bolender, 2007; Bolender et al., 2008).

As constituents of thoughts, concepts play a crucial role in accounting for the content of thoughts. The thought that there are dogs is distinct from the thought that there...
are flying dogs. A person who believes that there are dogs is writing this paper. A person who believes that there are flying dogs is likely to be in need of some assistance. Clearly, these two people are not having the same kind of thought. What makes these two thoughts distinct types of thoughts is the fact that they contain distinct concepts: the concept DOG and the concept FLYING DOG. There is more to the concepts DOG and FLYING DOG than them being distinct types of concepts. If an animal is capable of mentally representing the concept FLYING DOG it seems reasonable to expect this animal to also be capable of mentally representing the concept DOG. Yet, one might imagine an animal that can mentally represent the concept FLYING and the concept DOG without being able to mentally represent the concept FLYING DOG. That is because, an animal that possesses the concepts FLYING and DOG may lack the mental operation responsible for generating the concept FLYING DOG from these concepts. In fact, it is quite likely that the mental operation that enables the generation of concepts associated with entities that are not within experiential domain, i.e. Lambda Abstraction as we elaborate below, is unique to humans, an aspect of human uniqueness.

While our focus in this paper is the nature of uniquely human concepts, this is no excuse to assume that the conceptual systems of other animals are uniform. Results of an experimental paradigm known as reversal learning provide some evidence for possibly qualitative differences between the conceptual representations of distinct primate species (De Lillo & Visalberghi, 1994; Rumbaugh, 1971; Rumbaugh & Pate, 1984). Such experiments test the flexibility of animal responses to stimuli associated with varying reward regiments. Typically, the animals under test are first taught to discriminate between two stimuli, only one of which is rewarded (say, GREEN → Reward, BLUE → No Reward). After this learning step is established with some pre-determined accuracy, there is a complete reversal in reward contingencies (where GREEN → No Reward, BLUE → Reward). In a set of experiments on different primate species, Rumbaugh and colleagues have found that compared to great apes, prosimians have much more difficulty in inhibiting the previous associations and learning the new ones (Beran et al., 2008; Rumbaugh, 1971; Rumbaugh & Pate, 1984) Unlike prosimians, which employ a direct stimulus-response associational learning, great apes seem to take “a more cognitive, rule-based approach to the task, freed somewhat from the constraints of inhibitory and excitatory associations formed between specific stimuli and responses” (Beran et al., 2008, p. 5). In explicating

1) We take concepts to be mental entities. In the context of this paper, whether they have internal structure (Jackendoff, 1983, 1989) or not (Fodor, 1998) is a related but secondary issue so long as they are compositional. See the introductory chapter of Margolis and Laurence (1999) for a lucid overview of how (and to what extent) different theories of concepts handle compositionality.

2) For a review of experimental research using reversal learning tasks and the neural basis of reversal learning see Izquierdo et al. (2017).

3) In the experiments of Rumbaugh and colleagues, the stimuli that were linked with reward were junk toys rather than color cues.
what is cognitive about the approach that great apes adopt, Hurford (2007, p. 5) suggests that they associate the reward not directly with the stimulus as prosimians do but with a mental representation that contains the relevant concept. If the reward is associated with the relevant stimulus being of a certain color (say, GREEN), the mental representation associated with the reward before the reversal learning can be expressed as in (1):

\[ \text{is}(x, \text{GREEN}) \]

where \( x \) is a variable for an entity (the stimulus), \( \text{GREEN} \) is a concept and “is” is a term of relation which denotes the fall-under relation between the variable \( x \), here a unit of perception in the environment, and the concept \( \text{GREEN} \), where the entity that corresponds to the variable \( x \) falls under the concept \( \text{GREEN} \). Having a mental representation like (1) is useful for reversal learning in that, after the reward contingencies are reversed, instead of extinguishing a direct association between the stimulus and the reward and establishing a new association, what great apes do is associate the reward with the negation of the mental representation of (1), which is given in (2):

\[ \text{not}(\text{is}(x, \text{GREEN})) \]

This symbolic capacity is “a way of offloading redundant details from working memory”, making the task at hand achievable “without having to hold all the details in the mind.” (Deacon, 1997, p. 89). In this way, the learning of the new reward contingency is facilitated by the previously entertained mental representation. This, Hurford suggests, accounts for the varying degrees of success among primates at the task of reversal learning.

Observe that the variable \( x \) in the formulas (1) and (2) is not bound by any operator. Such free variables, we assume, obtain their semantic value via assignment functions which map them into entities typically in the perceived environment. In fact, building on Bolender (2007) and Bolender et al. (2008), we shall suggest that humans have the mental capacity to form representations with bound variables, thanks to the rule of Lambda Abstraction, which makes it possible for them to entertain concepts that are not in the domain of their experience. Before we present the details of this claim, let us clarify our assumptions about the grammar of great apes, to be called System N below, which defines expressions such as (1) and (2) above. Looking at (1), we see that it is a kind of predication relation between the variable \( x \) and the concept \( \text{GREEN} \). What kind of concepts should we expect in the grammar system of non-human apes? Hurford (2007, pp. 7–8) notes that “[i]t seems reasonable to suppose that (apart from innate concepts), an animal only has concepts of those types of things that it has at some time perceived (and only some of those)”. That is, “although humans can be in mental states relating to fictitious objects such as unicorns, it seems most unlikely that any non-human animal could be in such a state, having had no actual experience of unicorns.
or other fictions” since “[a]part from any innate concepts, an animal’s concepts are learnt through experience, the gateway to which is perception.” Vonk and Povinelli (2006, pp. 364–365) gives this conjecture a name, i.e. The Unobservability Hypothesis, which they elaborate as follows:

In short, we explore the possibility that whereas many species form concepts about observable things, humans alone think about such things as God, ghosts, gravity, and other minds. Further we speculate that although thinking about unobservables is by no means the only way in which the human mind differs from other species, it may serve as the foundation for many of the fundamental differences between our behavior and that of our closest living relatives. We note from the outset that this proposal does not conflict with other proposals that stress the importance of language in determining human uniqueness. Indeed, although we do not explore this idea at length in this chapter, we suspect that the underlying “abstractive depth” that makes reasoning about unobservables possible co-evolved with natural languages.

They review a set of experiments about concept formation, theory of mind and physical causality whose results have been interpreted as requiring that we attribute non-human animals a capacity for entertaining concepts for unobservables. They conclude that, in each case, the relevant results have been over-interpreted (see also Penn et al., 2008 and related commentaries). In this paper, we presuppose that The Unobservability Hypothesis provides a more or less accurate description of the differences between human and non-human cognition (however, see footnote 4 for some qualifications concerning innate concepts). As we understand it, if a concept is instantiated by an external object that is within the experience domain of some animal A, we shall call it a concept of perception (per-C) for the animal A. In what follows, we assume that the kind of concepts that are

4) In a related paper, Povinelli and Vonk (2003, p. 157) raise the possibility that “the human mind may have evolved a unique mental system that cannot help distorting the chimpanzee’s mind, obligatorily recreating in it its own image”, which is why, they suggest, we are so keen on attributing the capacity to represent mental states, which are unobservable entities, to non-human animals. This position might seem a little extreme and it is probably not an unavoidable consequence of the Unobservability Hypothesis once the modular nature of human mind is taken into consideration. Bolender (2007, p. 395), who is also committed to a version of the Unobservability Hypothesis, suggests that “theory of mind may be handled by a specialized and relatively peripheral module, universal to the species, which works largely independently of the more central belief system. Whatever processes this module utilizes to attribute intentional states are presumably distinct from the processes most centrally involved in cognition by description”, the ability to cognize entities one has no acquaintance with. Taking such modular concepts to be innate, we summarize our working hypothesis as the claim that as far as non-human animals are concerned non-innate concepts are perception-bound. This approach may also account for the relative abstractness of concepts related to social relations (Fiske, 1991) and rudimentary numerical capacities (Dehaene, 1996) in non-human animals. Crucially, a human concept such as FLYING TORTOISE is neither innate nor perception-bound.
exploited in the grammar of non-human animals is concepts of perception (per-Cs) and that such concepts have the property that they are instantiated by entities within the experience domain of animals.

Given the discussion thus far, we can formalize the grammar that defines the type of expressions such as (1) and (2). This grammar, we shall call *System N*. The lexicon of System N contains a set of variables for entities, VAR, a set of perceptible concepts, per-C, the *is* relation between VAR and per-C as well as some terms of relations, REL, which denote relations between entities.

\[(3) \quad \text{System N} \]

**Lexicon:** \( \text{VAR} \cup \text{per-C} \cup \{\text{is}\} \cup \text{REL} \)

where \( x, y, \ldots \in \text{VAR}, \text{HORSE, BLUE, \ldots} \in \text{per-C}, \text{BEHIND, SEE, \ldots} \in \text{REL} \)

**Syntax:**
- If “x” is a VAR and “C” is a per-C, then “is(x, C)” is a sentence
- If “\(x_1\)”, “\(x_2\)” … “\(x_n\)” are variables and “\(R_n\)” is an \(n\)-place relation, then “\(R_n(x_1, x_2, \ldots x_n)\)” is a sentence.
- If “\(\varphi\)” and “\(\psi\)” are sentences, so are “\(\neg(\varphi)\)”, “\(\varphi \& \psi\)”, “\(\varphi \lor \psi\)” and “\(\varphi \rightarrow \psi\)”. 

Let us exemplify what can, and crucially cannot, be done within System N. Consider, for instance, an experiment in which the images in (4) are shown to an animal endowed with System N for the first time:

\[(4) \quad \bigcirc \bigotimes \bigcirc \]

Let us assume, moreover, that the animal under test associates the variable \(x\) to the dotted circle, the variable \(y\) to the crossed square and the variable \(z\) to the dotted square. This assignment function \(g\), associated with the list of images in (4), is given in (5):

\[(5) \quad g: \{x \rightarrow \bigcirc, y \rightarrow \bigotimes, z \rightarrow \bigcirc\}\]

Endowed with System N, this animal can mentally represent the thought that the entity corresponding to variable \(x\) is a circle and that it is dotted (6a). It can also represent the
thought that y is a square but it is not dotted (6b). Moreover, it can mentally represent the thought that the dotted circle and the crossed square are next to each other (6c) as well as the thought that the entity corresponding to variable x is not both square and crossed (6d).

(6)   a. is(x, CIRCLE) & is(x, DOTTED)
   b. is(y, SQUARE) & not(is(y, DOTTED))
   c. is(x, CIRCLE) & is(x, DOTTED) & is(y, SQUARE) & is(y, CROSSED) & NEXT-TO(x,y)
   d. not(is(z, SQUARE) & is(z, CROSSED))

There are also limitations to the types of thoughts that can be represented in System N. Looking at the images in (4), one might notice the absence of a crossed circle (i.e. ⊗). Could an animal endowed with System N entertain the thought that there are no crossed circles? On the assumption that such an animal has never seen a crossed circle before, the answer seems to be negative. In order to have the thought that there are no crossed circles, the animal must first possess the concept CROSSED CIRCLE. However, by assumption, there has been no entity within the experience of domain of this animal that instantiates this concept. That is, it is not a concept of perception and, as a result, it is not in the lexicon. Then, the only way to mentally represent the concept CROSSED CIRCLE is via composing the concepts CROSSED and CIRCLE (both of which are, by assumption, concepts of perception, per-Cs). However, System N has no syntactic mechanism to compose per-Cs into new concepts. Hence, a concept such as CROSSED CIRCLE is not within the domain of cognition for the animal under test if it is a species of System N.

Humans seem to be able to entertain concepts that go beyond their experience domain. As we have noted above, they may look at (4) and notice the absence of a crossed circle. They may discuss what a flying dog could do, what a trunkless elephant looks like or whether there are any purple tigers in the world. How do humans break the limitations posed by a system that is bound by concepts of perception? We are here to suggest that they can do so with the help of a mental grammar in which variables can be bound by the lambda operators. This is the topic of the next section.

2 Concepts and Computations

Bickerton (2007, p. 511) claims that there are two main outstanding questions about the evolution of language faculty: (1) How did symbolic units evolve? (2) How did syntax evolve? These two novelties, Bickerton writes, are “the most salient (as well as the most difficult) of the things any adequate theory of language evolution must account for.” Crucially, for Bickerton, the answers to these questions are distinct and
independent. That is, “[t]here is no reason to believe that the emergence of the two was either simultaneous or due to similar causes, and some good reasons for supposing the contrary”. This assumption of dissociation between symbolic units and computations is one of the motivations behind Bickerton’s claim that the transition from animal communication systems to human language involved a stage of proto-language that contained symbols but no grammar. Bolender (2007), on the other hand, suggests that the mental computations involved in the derivation of syntactic representations may have transformative consequences for humans’ conceptual abilities. Indeed, it is thanks to the mental computations involved in variable-binding that humans have the capacity to think about entities that they have no direct experience about, the capacity for cognition by description, as Bolender calls it, a mentalistic re-interpretation of Russell’s (1911) knowledge by description. The possible dependency of uniquely human concepts on mental computations lead Bolender et al. (2008, p. 132) to propose “a working methodology, namely to investigate syntactic computations as a means of understanding uniquely human concepts”, a research methodology that will be followed closely in this paper.

2.1 Lambda Abstraction as a Formal Operation

How do syntactic computations give rise to concepts that go beyond the domain of experience? We suggest that the rule of Lambda Abstraction, which is basically a mental operation responsible for binding free variables, is the key to understanding this transition. Intuitively what this operation does is to turn a sentence with a free variable and a concept of perception (e.g. is(x, RED)) into a concept of property (e.g. the property of being RED). As we elaborate below, this ability of turning open sentences (i.e. sentences with free variables) into concepts of property accounts for the human capacity to go beyond concepts of perception and entertain concepts that are not necessarily within the domain of experience.

We first need to make some basic observations about lambda calculus, the computational framework within which the rule of Lambda Abstraction is defined. Lambda calculus, a mathematical language intended to capture the notion of function application (Church, 1932), is ultimately a notation for expressing functions and operations with functions. Consider the formula in (7):

\( x^2 + x \)

What does (7) evaluate to? It depends on the value of x, more specifically on what the assignment function g assigns to x. If g assigns 1 to x, then (7) evaluates to 2 but if g assigns to 2 to x, then (7) evaluates to 6. The variable x here is a free variable so that it takes its value directly from the assignment function. It is possible to bind each token of the variable x with the lambda operator ‘\( \lambda \)’ as in (8):

\( \lambda x (x^2 + x) \)
The ‘λ’ operator “binds the variable x, guarding it, as it were, from outside interference” (Alama & Korbmacher, 2021), in this example, from the contingencies of the assignment function g. That is, by binding the tokens of variable x in (7), this operation enables us to abstract over the specific values that x might get from this or that assignment function. This is the sense in which the lambda operator induces variable binding. In effect, the lambda term in (8) is a representation of the function which maps an arbitrary argument to the sum of itself and its square. When a certain argument, here a natural number, is fed into the lambda term, the output is obtained by replacing each bound variable in the expression with this argument (an operation called λ-Reduction or Beta-Reduction).

In the next section, we shall explore how Lambda Abstraction as a mental operation generates new types of concepts.

2.2 Lambda Abstraction as a Mental Operation

We shall now focus on the transformative consequences of Lambda Abstraction on human concepts. Consider the sentence in (10), defined within System N, which was described in the previous section.

\[
\text{is}(x, \text{CROSSED}) \& \text{is}(x, \text{CIRCLE})
\]

This is a sentence of System N with the free variable x, whose value is determined by the assignment function g. This open sentence can be turned into a lambda term by the application of the rule of Lambda Abstraction.

\[
\begin{align*}
\lambda x. x^2 + x \\
\text{(8)}
\end{align*}
\]

That is to say, the lambda expression in (8) is ultimately a notational variant of the function f below.

\[
f: \mathbb{N} \rightarrow \mathbb{N} \text{ such that} \forall x \in \mathbb{N}, f(x) = x^2 + x
\]
\[ \lambda x. \text{is}(x, \text{CROSSED}) \land \text{is}(x, \text{CIRCLE}) \]

Semantically, this lambda term expresses a property, the property of being CROSSED CIRCLE.\(^7\) It is a function that holds for an entity just in case this entity falls under CROSSED and it also falls under CIRCLE. In this way, the lambda expression in (12) characterizes the set of entities that are crossed circles, i.e. the set in (13):\(^8\)

\[ \{x: \text{is}(x, \text{CROSSED}) \land \text{is}(x, \text{CIRCLE})\} \]

While the lambda term in (12) expresses a concept of property, it does not, as of yet, express a thought about this concept. There are two ways in which (12) can be turned into a sentence expressing a thought. First, we can apply a variable, which corresponds to an entity, as an argument to this lambda term and obtain (14)b:

\[ a. \ [\lambda x. \text{is}(x, \text{CROSSED}) \land \text{is}(x, \text{CIRCLE})](y) \]
\[ b. \ \text{is}(y, \text{CROSSED}) \land \text{is}(y, \text{CIRCLE}) \quad \text{(}\lambda\text{-Reduction}) \]

Observe that such a thought could be expressed within System N and, therefore, does not really express a new type of thought made available by the rule of Lambda Abstraction. There is, however, a second way of turning (12) into a sentence: quantification. Indeed, it was Frege who claimed that quantifiers such as ‘something’ or ‘everything’ can be thought of as second-order concepts, which induce quantification over properties (Frege, 1879). Syntactically, they take lambda terms as argument and give sentences as output. Let us first note that P is a lambda term, then \( P^* \) is the set characterized by P (i.e. \( P^* = \{x: P(x)\} \)). The sentence formed by merging the quantifier ‘something’ with the lambda term P says that \( P^* \) is not empty. The merger of the quantifier ‘everything’ and the lambda term P say that \( P^* \) is the set of all things. Finally, if we have sets characterized by lambda terms, then we can count the cardinality of the members of such sets. This is what cardinality functions do. In the light of this discussion, we add existential, universal and cardinal quantifiers into our system. We shall call this new grammar System N+1 to highlight the fact that it contains (and in continuum with) System N.

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\(^7\) It does not sound implausible that such concepts interface with systems of vision giving rise to Mental Imagery, which is understood as “representations and the accompanying experience of sensory information without a direct external stimulus” (Pearson et al., 2015, p. 590) in fact perhaps without any external stimulus at all. For instance, the mental experience generated by the interfacing of the concept CROSSED CIRCLE with the system of Mental Imagery might be relevant for the validation of this concept in the absence of any external experience.

\(^8\) If P is a lambda term, then the set characterized by P, \( P^* \), is the set of things that P is true of. That is, \( P^* = \{x: P(x)\} \).
System N+1 contains System N and the rule of Lambda Abstraction and the following rules:

- If “P” is a lambda term and “x” is a variable, then “P(x)” is a sentence.
- If “P” is a lambda term, then “∃(P)” is a sentence, where “∃(P)” means P* ≠ ∅.
- If “P” is a lambda term, then “∀(P)” is a sentence, where “∀(P)” means P* = U.
- If “P” is a lambda term and “n” is a numeral, then “n(P)” is a sentence, where “n(P)” means |P*| = n.

Recall from the previous section the experimental set-up in which an animal was looking at the screen containing the items in (16), repeated from (4).

(16) ⊙ ⊠ ⊡

In the previous section, we have noted that looking at (16), an animal endowed with System N cannot form the sentence expressing the thought that there are no crossed circles. Interestingly an animal endowed with System N+1 can form the sentence in (17) expressing exactly this thought.

(17) not(∃(λx. is(x, CROSSED) & is(x, CIRCLE))

As expected, this sentence holds just in case {x: is(x, CROSSED) & is(x, CIRCLE)} is empty. Once the rule of Lambda Abstraction was available, it became possible to generate concepts of property. In this way, a species of System N+1 can entertain mental concepts of flying dogs (λx. is(x, FLYING) & is(x, DOG) or purple tigers (λx. is(x, PURPLE) & is(x, TIGER)). In fact, such an animal can entertain the concept of trunkless elephants without having to see one.

We now have a new concept, the concept of PURPLE TIGER, built out of two concepts of perception: PURPLE and TIGER. This operation may look similar to Pietroski’s (2018) M-Join, which takes two monadic concepts to generate a new one, i.e. PURPLE(_, TIGER(_)), where the blanks indicate the adicity of these concepts and the carrot symbol indicates their conjunctive composition. A hypothesis that is in line with the general outlook of this paper could be that while humans share basic predicative concepts with many other animals (see Pietroski, 2018, p. 28 for an explicit remark in favor of this position), the capacity to apply M-Join to basic concepts to generate a new one may be uniquely human.

We wish to point out that, whatever the merits of this approach might be, it is incompatible with the hypothesis being explored in this paper. First, we take conjunctivity, which is what M-Join is intended to achieve, to be a relatively basic property of human grammar, ultimately a relic of an earlier system. In fact, in Section 3, following recent work on the syntax and semantics of connectives, we shall argue that there is only one mode of conjunctivity in human grammar and that is sentential coordination (Hirsch, 2017), a rule borrowed from System N. That is, we do not find much novelty in conjunctivity. Secondly, we make a categorical distinction between concepts of perception and concepts of property. Concepts of perception have no adicity, cannot take arguments and cannot be quantified over since they are not of the right type. Concepts of property, on the other hand, can be derived at the logical form, can take arguments (in syntactic terms, they have edge features, Chomsky, 2008) and they can be quantified over. We
(18) \( \lambda x. \text{is}(x, \text{ELEPHANT}) \& \neg(\exists(y. \text{is}(y, \text{TRUNK}) \& \text{HAVE}(x,y))) \)

which characterizes the set in (19):

(19) \( \{x: \text{is}(x, \text{ELEPHANT}) \& \{y: \text{is}(y, \text{TRUNK}) \& \text{HAVE}(x,y)\} = \emptyset\} \)

One aspect in which concepts of property differ from concepts of perception is that they can be quantified over. In a sense, the availability of the rule of Lambda Abstraction in System N+1 has unlocked the potential for quantification. This is of significance in that it is through quantification that concepts of property can be turned into new types of thoughts, thoughts that are about concepts whose instantiations are not necessarily within the domain of experience. Thus far, we have focused on sentences involving existential quantification. What about sentences with more complex logical forms? Russell, for instance, argued that a sentence involving a definite description (i.e. a sentence typically of the form \( \text{the } F \text{ is } G \) where \( \text{the } F \) is the definite description) has a multiclausal logical form. Looking at a copy of the book \( \text{Sir Gawain and the Green Knight} \), we may entertain the thought that the author of this book is dead even though apparently nobody knows who wrote it. We know the author of \( \text{Sir Gawain and the Green Knight} \) not by having any direct experience involving this person but only by a description of this person, what Russell (1911) called knowledge by description. In Russell’s analysis, sentences with definite descriptions, which have logical forms with uniqueness (i.e. one and only one person wrote this book) and universal quantification (i.e. whoever wrote this book is dead.), are involved in this mode of knowing. The logical form of the sentence “The author of this book is dead”, as defined within System N+1, is shown below:

(20) (Uniqueness) one(\(\lambda x.\text{AUTHOR}(x,y) \& \text{is}(y, \text{BOOK})\)) &

(Universality) \( \forall(\lambda x.[\text{AUTHOR}(x,y) \& \text{is}(y, \text{BOOK}) \rightarrow \text{is}(x, \text{DEAD})) \))

This means that, under the analysis developed in this paper, the capacity to entertain thoughts that involve definite descriptions is a complex consequence of the capacity to employ the rule of Lambda Abstraction as well as the capacity to quantify over lambda-abstracted terms.

suggest that human grammar is unique in having concepts of property given that such concepts are generated by the rule of Lambda Abstraction.

10) We assume here that the concepts FLYING, DOG, PURPLE, TIGER, TRUNK, ELEPHANT etc. are concepts of perception. That is, there are entities in the perceptual environment of animals that fall under these concepts.

11) Later work has shown that the uniqueness aspect of sentences with definite descriptions is better analyzed as a presupposition (Strawson, 1950 and subsequent work), a condition that must be met for such a sentence to have a semantic value at all. We ignore this issue here.
Now that we have a sense of the differences between System N and System N + 1, we are in a position to state the central claim about this paper. We claim that humans and only humans have surpassed the limitations of a system of perceptual concepts (i.e. System N).

(21) The System N+1 Hypothesis
Only humans entertain mental representations constructed within System N+1.

The addition of the rule of Lambda Abstraction into System N made it possible for humans to entertain concepts of property which are not bound by experience. With the help of quantification, such concepts gave rise to new types of thoughts, thoughts about entities that are not within the domain of experience. Not surprisingly, then, we suggest that this capacity of humans is enabled by the fact that humans are a species of System N+1.

3 The Antiquity of Connectives
Looking at the history of sentential operators such as connectives, we find that they are, in a sense, relics of System N. Within this system, they are understood to be functions from sentences to sentences. In natural languages, however, such connectives seem to do more than just conjoin sentences as in (22a). It seems that they can, for instance, appear between verbal (22b) or nominal phrases (22c).

(22) a. \([\text{Sue saw John}] \text{ and } [\text{Bill kissed Mary}]\).
   b. Sue \(\text{[VP saw John]} \text{ and } [\text{VP kissed Bill}]\).
   c. Sue saw \(\text{[NP John]} \text{ and } [\text{NP Bill}]\).

One line of research suggests that the availability of nominal and verbal coordination requires that the connective and have variable lexical entries in each structure, a type of ambiguity, that go beyond its simple sentential function. That is, we need to assign distinct lexical entries to the connective and for its sentential, verbal and nominal uses typically with the help of additional rules (Jacobson, 1999; Partee & Rooth, 1983). An alternative approach, one that is in line with the proposal developed in this paper, claims that the appearance of ambiguity for the connective and is a consequence of the fact that there is more structure to (22b) and (22c) than meets the eye. Indeed, it is by now standard within Minimalist Syntax to analyze subjects as having an initial low position within the verb phrase (The VP-Internal Subject Hypothesis, Koopman & Sportiche, 1991; Speas, 1986; Woolford, 1991). That is to say, the subject Sue in (22b) originates within each VP and then moves across the board to its sentence initial position. The underlying syntactic representation of (22b) is shown in (23), where the traces that are left behind by the moved subject are indicated with identically indexed traces.
(23) Sue₁ [[VP₁ t₁ saw John] and [VP₁ t₁ kissed Bill]].

The traces in (23) correspond to a variable at the level of logical form. The logical form of the coordination of verbal phrases is shown in (24), where the variable x replaces the trace indexed with the numeral one. Observe that this expression is a coordination of two sentences, as expected if connectives are always sentential operators.\(^\text{12}\)

(24) SAW(x, j) & KISSED(x, b)

The logical form of (23) is obtained via the application of the rule of Lambda Abstraction to (24). The argument of the resulting lambda term is the subject.

(25) \([\lambda x. \text{SAW}(x, j) \& \text{KISSED}(x, b)](s)\)

What about nominal coordination in (22c)? Hirsch (2016, 2017) makes a set of observations suggesting that what looks like nominal coordination is actually sentential coordination. One piece of evidence for this claim comes from the distribution of temporal adverbs. Typically, a temporal adverb like yesterday cannot be used to modify a nominal phrase (Hirsch, 2017, p. 83).\(^\text{13}\)

(26) a. ?? John saw yesterday Chomsky.
   b. ??* John saw yesterday me.
   c. ??* John flew off to yesterday Paris.

Inside nominal coordination, however, the use of the temporal adverb yesterday becomes acceptable (Hirsch, 2017, p. 83).

(27) a. John saw Labov and yesterday Chomsky.
   b. John saw me and yesterday you.

This pattern of judgments can be accounted for on the assumption that temporal adverbs cannot modify nominal phrases and that what yesterday modifies in (27a–c) is not a nominal phrase but a hidden sentence. Under this analysis, the syntactic representation of (27a) would be as in (28):

---

12) For reasons of brevity, we analyze names as denoting individuals throughout the paper. It is worth mentioning that this is far from obvious. See Abbott (2002), Fara (2015), Geurts (1997), Kripke (1972), and Matushansky (2008), among others, for philosophical and linguistic aspects of referentialism vs predicativism debate about names.

13) The symbols “??” and “??*” are intended to expresses different degrees of unacceptability where a sentence prefixed with “??*” is more unacceptable than a sentence prefixed with “??”. (To report that a certain string of words is completely unacceptable, linguists prefix it with an asterisk “*”)
(28) \[
\text{[John} \text{, } [\text{VP } t_1 \text{ saw Labov}] \& [\text{VP yesterday } [\text{VP } t_1 \text{ saw Chomsky}]]] \]

The deletion of the verb saw in the second conjunct is the consequence of a productive process in English known as gapping (Jackendoff, 1971; Ross, 1970, for a recent overview see Johnson, 2019) in which the verb (saw in (29a)) or the verbal complex (will see in (29b)) in the second conjunct gets to be deleted under identity.\(^{14}\)

(29) a. John saw Bill and Mary Sue. (from John saw Bill and Mary saw Sue.)

b. John will see Bill and Mary Sue. (from John will see Bill and Mary will see Sue.)

All in all, there is reason to believe that connectives in natural languages conjoin sentences and sentences only. When they appear to conjoin nominal or verbal phrases, we find that such structures have an underlying sentential syntax, which can be diagnosed with the help of the distribution of adverbs. We claim that the sentential nature of connectives is a direct consequence of the fact that they are relics of System N, in which they function as sentential operators.

\(^{14}\)There is additional independent evidence for the presence of a silent verb (or verbal complex) in cases that look like nominal coordination. As Hirsch (2017) notes, the silent verb in gapping constructions plays a crucial role in licensing what is known as VP-ellipsis in English, examples of which are given in (i) and (ii) (for an overview of such constructions, see Johnson, 2001).

(i) Sue read War and Peace and Bill did ∆ too.

(ii) Sue will read War and Peace and Bill will ∆ too.

In each case, the elided material is represented with the symbol ∆, where ∆ is identical the VP of the first sentence, i.e. [VP read War and Peace], sometimes called the antecedent VP. For VP ellipsis to be possible in an out-of-blue context, there must be an antecedent VP in the linguistic structure that licenses it. Consider now (iii) from Hirsch (2017, p. 68):

(iii) Harvard invited Labov and, ten years after Brandeis did ∆, Chomsky.

Here the elided VP, represented with ∆, corresponds to [VP invite Chomsky]. This means that for (iii) to be felicitous in an out-of-blue context, there must an antecedent VP that licenses the application of VP-ellipsis. Hirsch (2017) observes that this condition can be met if we assume that there is a gapped verb in the second conjunct whose syntactic presence enables the ellipsis operation by providing an antecedent VP for ∆ in (iii), as shown in (iv).

(iv) Harvard invited Labov and, ten years after Brandeis did ∆, [VP invited Chomsky].

This analysis provides independent evidence in favor of a gapping analysis for the apparent conjunction of nominal phrases.
4 The Naturalness of Lambda Abstraction

We have argued that Lambda Abstraction enables mental representation of concepts that can be used for entities that might not be within the domain of human experience. In this section, we shall focus on the relation between the syntactic representation of a linguistic expression and the application of Lambda Abstraction at the level of logical form. Following much research on relative clauses (Heim & Kratzer, 1998 a.o.), we suggest that such clauses correspond to lambda terms at the level of logical form. Indeed, more generally, constructions involving movement chains, one of which is relative clauses, require the application of the rule of Lambda Abstraction for reasons of interpretability, as we shall discuss. This raises the possibility that there is a one-to-one correspondence between the application of the movement operation (i.e. sometimes called “Internal Merge”) in syntax and variable binding at logical form, an analytical possibility defended in Bolender, 2007; Bolender et al., 2008 in the context of quantificational binding. We suggest, however, that there are also cases in which Lambda Abstraction is required in the absence of any movement operations. Therefore, we claim that the rule of Lambda Abstraction applies at logical form whenever doing so leads to an interpretable structure. The presence of a movement chain presents one such case but the application of the rule of Lambda Abstraction is not contingent on the availability of the movement operation in syntax.

Quine (1964, p. 109) notes that “the peculiar genius of the relative clause is that it creates from a sentence “…x…” a complex adjective summing up what that sentence says about x.” Syntactically, this is achieved via the operator movement of the relative pronoun to a sentence initial position (Chomsky, 1977). Given the sentence in (30a), for instance, we can generate the relative clause in (30b) via the movement of the relative pronoun whom, which leaves a trace in its initial position. This relative clause can then be used to modify a noun as in (30c).

(30)   a.  [John saw whom]
       b.  … whom₁ [John saw t₁]
       c.  … [[boy] [whom₁ [John saw t₁]]]

The LF consequence of the operator movement is that it creates a lambda term from an open sentence making use of the rule of Lambda Abstraction. As a consequence of the co-indexation between the relative pronoun and the trace, the variable associated with the trace is bound by the lambda operator. That is, the logical form of (30b), shown below as (31b), is obtained by the application of the rule of Lambda Abstraction to the

15) We take logical form to be an interface level between narrow syntax and the conceptual-intentional system. It maps syntactic objects into semantically interpretable syntactic objects. Generating semantically interpretable syntactic structures is the main function of the rule of lambda abstraction, for instance.
open sentence in (31a). The resulting lambda term can now be used to modify the noun
boy:

\[
\begin{align*}
(31) & \quad \text{a. LF(John saw } t_1) = \text{SAW}(j, x) \\
& \quad \text{b. LF(whom}_1 \text{ John saw } t_1) = \lambda x. \text{SAW}(j, x) \\
& \quad \text{c. LF(boy whom}_1 \text{ John saw } t_1) = \lambda x. \text{is}(x, \text{BOY}) \& \text{SAW}(j, x)
\end{align*}
\]

This discussion indicates that there is something natural about the rule of Lambda
Abstraction. More specifically, there is a syntactic operation, i.e. the operator movement,
which directly translates into the application of Lambda Abstraction at logical form.
This is not to say that the application of this rule is contingent on the availability
of the operator movement. We have seen, in the previous section, that there is a wide
consensus within Minimalist Syntax that subjects originate inside the verb phrase and
that their final position is a consequence of the movement of the subject to the sentence
initial position.

\[
\begin{align*}
(32) & \quad [\text{John}_1 \left[ \text{VP}_1 \text{ saw Mary} \right]]
\end{align*}
\]

Interestingly, the resulting structure in (32) is not interpretable in the absence of Lamb-
da Abstraction. The logical form of the VP in (32) corresponds to the open sentence
\( \text{SAW(x,m)} \) which contains a free variable. We do not have any syntactic rule that takes an
open sentence and a name and gives us a sentence, which can then be associated with a
meaning. For (32) to have an interpretation, we need to assume that the open sentence is
turned into a lambda term via the application of the rule of Lambda Abstraction.

\[
\begin{align*}
(33) & \quad \lambda x. \text{SAW(x, m)}
\end{align*}
\]

The logical form of (32), under these assumptions, can be representation as in (34).

\[
\begin{align*}
(34) & \quad [\lambda x. \text{SAW(x, m)}](j)
\end{align*}
\]

What this means is that it is not only the operator movement that conditions the
application of the rule of Lambda Abstraction but other types of movement operations
can do so as well. Schematically, we can represent the relation between the movement
operation and Lambda Abstraction as in (35), where \( \alpha \) is the element that has moved out
of \( \beta \). We assume that the operator movement of the relative pronoun does nothing more

---

16) The algorithm responsible for translating co-indexation in syntax to lambda binding at logical form is a complex
one. We will give a schematic recipe in (35). For a more step-by-step treatment, see Heim and Kratzer (1998, Chapter
5).

17) The function LF, short for logical form, in (31) maps syntactic structures into lambda expressions. It is a function
from narrow syntactic objects to their logical forms.
than inducing the application of the rule of Lambda Abstraction. That is, the relative pronoun itself is semantically vacuous and has no corresponding expression at the level of logical form.

(35) SYNTAX: \[ \alpha, \beta[t] \]
LOGICAL FORM: \[ \lambda x. \beta[x] \] if \( \alpha \) is a relative pronoun
\[ [\lambda x. \beta[x]](\alpha) \] if \( \alpha \) is a name\(^{18}\)

It looks like there is a fairly direct link between the movement operation in syntax and the rule of Lambda Abstraction at logical form. Given that Lambda Abstraction induces variable binding, we can say that there seems to be a close link between movement in syntax and variable binding at logical form. This claim comes very close to Bolender’s suggestion that movement in syntax results in variable binding, which plays a crucial role in “the ability to conceive of things one had never experiences, such as mythological beings, places only visited by the dead, and so forth.” (Bolender, 2007, p. 384). Under Bolender’s analysis, there is a one-to-one relationship between quantification and variable-binding in that it is through quantifiers that variables are bound (as in first order logic). In this paper, we dissociate variable binding from quantification with the introduction of the rule of Lambda Abstraction (see Heim & Kratzer, 1998, Section 7.4.1 on an explicit discussion of this separation). Lambda Abstraction enables the mental representation of new kinds of concepts, concepts whose instantiations need not be within the domain of experience, and we assume that the representation of such concepts is prior to making quantificational claims about such concepts. As we have said before, we take it that Lambda Abstraction has unlocked the potential for quantification by defining a new type of linguistic object: lambda terms.

\(^{18}\) A third LF rule concerns quantifiers such as *something* and *everything*. Given the syntax in (35), the relevant LF rule would be as follows:

(i) \( \alpha(\lambda x. \beta[x]) \) if \( \alpha \) is a quantifier

This rule is relevant for sentences involving a quantifier such as (ii), which, we shall assume, involve quantifier raising as in (iii)

(ii) \[ \text{[John ate something]} \]

(iii) \[ \text{Something}_1, [\text{John ate } t_1] \]

The LF rule above tells us that the logical form of (iii) is as in (iv):

(iv) \( \exists(\lambda x.\text{ATE}(j,x)) \)
This being said, it seems that there are cases where we need to dissociate syntactic movement from Lambda Abstraction, hence syntactic movement from variable binding. Consider a relative clause modifying a noun:

(36)  \[\text{NP} \ [\text{NP} \ \text{dog}_1] \ [\text{RelC} \ \text{which}_1 \ t_1 \ \text{flies}]\]

The logical form of this sentence should characterize the set of dogs that fly as shown in (37):

(37)  \{x: \text{is}(x, \text{DOG}) \ & \ \text{FLIES}(x)\}

This requires that we first conjoin the open sentence containing the concept DOG and the open sentence obtained via the application a variable to the lambda term corresponding to the relative clause.\(^{19}\)

(38)  \text{is}(x, \text{DOG}) \ & \ [\lambda y. \text{FLIES}(y)](x)

We can now apply the rule of Lambda Abstraction in order to obtain the function that characterizes the set in (37).

(39)  a.  \(\lambda x. \text{is}(x, \text{DOG}) \ & \ [\lambda y. \text{FLIES}(y)](x)\)
    b.  \(\lambda x. \text{is}(x, \text{DOG}) \ & \ \text{FLIES}(x)\)  \hspace{1cm} \text{(After applying the variable x to}  \ [\lambda y. \text{FLIES}(y)]\)

This is, indeed, the lambda term that characterizes (37). Crucially, however, there was no movement operation that triggered the application of the rule of Lambda Abstraction. That is to say, variable binding does not seem to be contingent on the availability of movement in syntax.

Observe that the lambda term in (39b) can now function as input to quantifiers. For instance, the logical form of the sentence in (40a) can be represented as (40b):

(40)  a.  There is no dog which flies
    b.  \(\text{LF}(40a) = \text{not}(\exists(\lambda x. \text{is}(x, \text{DOG}) \ & \ \text{FLIES}(x)))\)

All in all, while movement in syntax seems to be sufficient for inducing variable binding at logical form, it is not, strictly speaking, necessary. We summarize this observation in the following principle:

---

\(^{19}\) The identity of the variables in \text{is}(x, \text{DOG}) and \text{FLIES}(x), i.e. the fact that they both contain the variable x, can be thought to be a consequence of the co-indexation between the noun and the relative pronoun, as can be seen in (36). Making this idea explicit is not a trivial matter. In Heim and Kratzer (1998), this identity is achieved with the rule of predicate modification.
The Principle of Lambda Interpretability

Apply the rule of Lambda Abstraction whenever doing so leads to an interpretable expression.

It is not surprising, then, that there are instances of Lambda Abstraction at logical form in the absence of any movement operation in syntax.

5 The Implementation of Lambda Abstraction in the Brain

We have argued that the human capacity to entertain concepts for entities that go beyond the experiential domain is a consequence of the fact that humans are a species of System N+1, hence capable of generating and processing mental representations involving Lambda Abstraction. We shall now briefly discuss the brain mechanisms that might be involved in the representation and execution of Lambda Abstraction. While a fully explicit hypothesis as to how System N+1 is implemented in the brain is beyond what this paper can offer, we find it important to highlight the significance of some of the brain functions (and the brain regions associated with them) for the generation and/or processing of linguistic expressions defined by System N+1. We first show that Lambda Abstraction, when applied to event variables, provides a formal account of the human capacity for event conceptualization. Interestingly, there is experimental evidence that the hippocampus, located deep in the temporal lobes, is crucially involved in event conceptualization (Hassabis et al., 2007), from which we conclude that the hippocampus has some role to play in interpreting lambda-abstracted terms. Secondly, we observe that the execution of Lambda Abstraction requires access to the derivational history of the structure being built, one aspect in which this rule is similar to Internal Merge, a subcase of the structural building operation Merge in Minimalist Syntax. It is hypothesized that procedural memory plays an important role in the execution of mental operations that scan earlier stages of a derivation (Bolender et al., 2008; Piattelli-Palmarini & Uriagereka, 2005), from which we conclude that Lambda Abstraction is contingent on the availability of enhanced procedural memory.

5.1 Event Conceptualization and the Hippocampus

In this section, we shall focus on the capacity for event conceptualization and the role that the hippocampus plays in supporting this capacity. We show that event conceptualization is a cognitive consequence of Lambda Abstraction being applied to free event variables. To do so, we must first convince ourselves that event variables exist in human grammars and that they can, in principle, be abstracted over. Consider (42a) and (42b) from Krifka (1990):

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a. Four thousand ships passed through the lock last year.
b. The library lent out 23,000 books in 1987.

Interestingly, we are not really counting the set of ships in (42a) or the set of books in (42b). When we know that (42a) is true, we do not really know how many distinct ships were involved in the last year’s passing events. It is possible that the same ship passed more than once, in fact perhaps many times. Similarly knowing that (42b) holds does not tell us anything about the cardinality of the set of books that the library lent out in 1987. What we are counting in (42a) is the set of events which involves the passing of a ship. Similarly, in (42b) we are given information about the cardinality of the set of events in which the library lent out a book in 1987. That is, it is not entities we are counting but events. This means that we must allow event variables into our grammar and allow abstraction over them to obtain sets of events, which can then be quantified over and counted.

Now that we have established the presence of event variables, let us focus on the hippocampus and the role it plays in event conceptualization. Duff and Brown-Schmidt (2012, p. 5) note that in addition to its role in the formation of long-term episodic memories and in spatial cognition, the hippocampus plays a crucial role in relational binding, which involves “the binding of the arbitrary co-occurrences of people, places and things of a scene or an event.” Such operations “link the spatial, temporal and interactional relations” relevant for an event. It seems likely that relational binding is involved in the processing of logical forms (with event variables) by the external conceptual systems. For instance, walking along the Bosporus Strait in a mildly cold İstanbul night, one may come across the unfortunate event of a cat biting Ahmet. This is an event of biting whose agent is a cat, whose patient is Ahmet and which took place in Istanbul and at night. The symbolic representation of this event, as defined within System N with event variables, can be shown as in (43):

\[
(43) \quad \text{BITING}(e) \land \text{AGENT}(e, x) \land \text{is}(x, \text{CAT}) \land \text{PATIENT}(e, \text{Ahmet}) \land \text{TIME}(e, \text{night}) \land \text{PLACE}(e, \text{Istanbul})
\]

The processing of this logical form clearly involves relational binding, supported by the hippocampus, given that it requires bringing different aspects of a single event (its participants, its time and its place) together.

There is more to the hippocampus than just relational binding. Reporting on the results of an experiment in which patients with amnesia associated with bilateral hippocampal damage were compared to a controlled group in their ability to construct imaginary experiences, Hassabis et al. (2007) note that patients with hippocampal amnesia show deficit in imagining novel experiences. They suggest that “the role of hippocampus extends beyond reliving past experiences, encompassing ... the construction of fictitious experiences.” (p. 1729). It seems plausible that the capacity to construct a fictitious
experience requires the capacity to mentally construct a type of event that one has not experienced before. Let us call this capacity event conceptualization and ask what is formally involved in it. Recall the unfortunate biting event involving Ahmet. The observer of this event might wonder, if they are a species of System N+1 as we shall see, whether there has been any event in which a man bit a cat. Such a query would involve the conceptualization of an event in which the agent is an arbitrary man, the patient is an arbitrary cat, a type of event that, in all likelihood, the observer has never experienced before. This means that the logical form associated with an event of this kind should not contain a free variable linked to the event of biting in the perceived environment. Rather, the event variable in this logical form must be bound by the lambda operator so that it is protected from the contingencies of this or that assignment function. More specifically, the logical form associated with this event-concept involves abstraction over the event variable with the help of the lambda operator, as shown in (44):

\[
\lambda e. \text{BITING}(e) \land \exists(\lambda x. \text{is}(x, \text{MAN}) \land \text{AGENT}(e, x) \land \exists(\lambda y. \text{is}(y, \text{CAT}) \land \text{PATIENT}(e, y)))
\]

We observe that the mental representation of such an event-concept requires the tools made available by System N+1. Assuming that only humans entertain mental representations constructed within System N+1 (The System N+1 Hypothesis in (21)), we put forth the hypothesis that that only humans are capable of event conceptualization, a capacity that is supported by the hippocampus. Summarizing research on the evolution of the hippocampus in the primates (Barger et al., 2014; Xu et al., 2018), Benítez-Burraco (2021, p. 36) notes that “it seems that changes in the hippocampus, both quantitative and qualitative, have occurred during human evolution that account for an important portion of the modifications experienced by the human brain.” A hypothesis that is line with what we have said so far in this paper is that these changes in the hippocampus during human evolution have played a crucial role in the human capacity for event conceptualization, which we take to be a transformative consequence of humans being a species of System N+1.

5.2 The Role of Procedural Memory in Lambda Abstraction

In this section, we shall focus on the interaction between procedural memory and the execution of the rule of Lambda Abstraction. Within the declarative/procedural model of memory (Ullman, 2001a, 2004; Ullman & Pierpont, 2005), different components of language interact selectively with different types of memory. Lexicon, the storage of basic units of computation, typically morphemes together with all the syntactic, semantic and phonological information associated with them, relies on declarative memory. Procedural memory is involved in the execution of the rules of symbol manipulation responsible for the productivity of language, i.e. grammar, as well as in the execution of motor skills.
In Minimalist Syntax, the only structure building operation is Merge, which takes two syntactic objects as its arguments and generates a new syntactic object that contains these two arguments as its sole members (Chomsky, 2013; Collins, 2002; Seely, 2006).

\[
\text{Merge}(X, Y) = \{X, Y\} \text{ (X and Y are simple or complex syntactic objects)}
\]

Merge has two subcases that must be distinguished from each other (Chomsky, 2004). When merging two syntactic objects \( X \) and \( Y \), \( X \) is either contained in \( Y \) or it is not contained in \( Y \). When \( X \) is not contained in \( Y \), this operation is referred to as External Merge (as \( X \) is external to \( Y \)). When the auxiliary HAVE in English is merged with the verb phrase containing the verb seen and its arguments John and Mary, this is an instance of External Merge since the auxiliary is not to be found anywhere inside the verb phrase.

\[
\text{Merge(HAVE, \{Mary, \{seen, John\}\}) = \{HAVE, \{Mary, \{seen, John\}\}\}}
\]

The morphological form of this auxiliary, which carries the tense information about the sentence, depends on the person and number features of the subject. In technical terms, the auxiliary HAVE has unvalued person and number features (unvalued \( \phi \)-features, for short) that must be valued by a nominal phrase within its sister constituent. To this end, the auxiliary scans the constituents inside the verb phrase until it finds a nominal phrase that has valued \( \phi \)-features, here the subject Mary. Once the unvalued features on HAVE are valued as third person singular by the \( \phi \)-features on Mary, its morphological form can be established as has. In return, the auxiliary assigns nominative to the unvalued case feature of Mary. This agreement operation (AGREE of Chomsky, 2000, 2001) between the auxiliary and the subject is a pre-condition for the re-merge of Mary with the phrase headed by the auxiliary. In this operation, shown in (47), a copy of Mary is already contained in the phrase with which it merges, hence this is a case of Internal Merge. The lower copy of Mary is deleted at the phonological component, as a result of which the output of Merge in (47) is pronounced as Mary has seen John.

\[
\text{Merge(Mary, \{has, \{Mary, \{seen, John\}\}\}) = \{Mary, \{has, \{Mary, \{seen, John\}\}\}\}}
\]

Observe that the condition for the application of Internal Merge is quite involved when we consider the type of information that must be available for its execution: A lexical element with unvalued features ends up scanning earlier stages of the derivation to find an appropriate phrase to agree with, a condition that is not relevant for External Merge. Such a scan requires an “on-line ability to record what took place in a particular derivation” (Piattelli-Palmarini & Uriagereka, 2005), an access to derivational history. This requirement associated with Internal Merge puts high demands on procedural memory (Bolender et al., 2008). Given this discussion, we find that there should be extensive
involvement of procedural memory in the execution of Lambda Abstraction, too. That is because, this rule requires access to derivational history in order to find a free variable to abstract over. More specifically, the rule of Lambda Abstraction takes a sentence as an input, scans this sentence to find the variables in it and asks for each variable found in this scan whether it is bound or not. Once a free variable is found, the lambda operator that binds it is introduced to the structure. As an example, consider the lambda term associated with the concept ONE WHO IS LOVED. This concept is obtained by applying the rule of Lambda Abstraction to the sentence in (48a). First, the sentence is scanned to obtain a list of variables in it (here, \(x\) and \(y\)). For each variable, it must be determined whether this variable is bound or not (here, \(x\) is bound, but \(y\) is not). Then, the lambda operator targets the free variable to obtain the concept in (48b). All in all, we see that there are some formal similarities between Internal Merge and Lambda Abstraction when it comes to the type of memory their execution requires.

(48)  
a. \(\exists(\lambda x.\text{LOVES}(x,y))\)  

(b) \(\lambda y.\exists(\lambda x.\text{LOVES}(x,y))\)

This might be a good point to compare the main proposal of this paper to the similar ideas developed by Bolender (2007) and Bolender et al. (2008). We have seen that there is an asymmetry between External Merge and Internal Merge when it comes to the need to access derivational history. Building on this observation, Bolender and colleagues suggest that while Internal Merge might logically be a subcase of Merge, its execution is contingent on the availability of enhanced procedural memory. Consequently, this operation remained as a dormant potential until quite recently in human evolution, presumably taking effect right before the transition to the Upper Paleolithic. Once implementable, Internal Merge gave rise to constructions involving variable-binding in the form of quantification, which, in return, made the capacity to cognize objects that one is not acquainted with, i.e. cognition by description, a possibility for the members of the human species. We agree that Internal Merge was not an immediate consequence of the Merge operation, but for a reason that is slightly different from what Bolender and colleagues claim. Recall from Section 4 that there are cases where Lambda Abstraction applies in the absence of Internal Merge. However, Internal Merge is simply uninterpretable in the absence of Lambda Abstraction (at least within the semantic framework with which we are working). In other words, it is the application of the rule of Lambda Abstraction, which also relies on enhanced procedural memory, that makes constructions involving Internal Merge interpretable at the interfaces. This rule enables humans to execute semantically consequential Internal Merge and to construct abstract concepts, two possibly unique aspects of human language. Moreover, syntactic objects this rule generates are inputs to quantification, another human feat. We, therefore, take it that Lambda Abstraction is crucially involved in the account of the unique properties of human language faculty,
ultimately human cognition. Moreover, we have seen that when a syntactic object is merged in two distinct position (i.e. Internal Merge) the higher copy is interpreted as an operator and the lower copy as a variable bound by this operator. How so? How are these two copies interpreted in the way that they are? Lambda Abstraction makes the mechanism behind variable binding formally explicit, taking it to be an interface rule applied at the logical form rather than narrow syntax.

Let us finally note that the proposal developed in this paper makes an explicit claim about the bits and pieces of primate grammar, i.e. System N. Basically, it is some kind of Predicate Logic with variables but with no quantifiers. As a result, our analysis of the transition from System N to System N+1 highlights both continuity and discontinuity. The continuity in this transition is deducible from the fact that System N+1 contains System N. However, only System N+1 grants its users the capacity to generate concepts for entities beyond their domain of experience, a clear instance of discontinuity.

6 Discussion and Conclusion

There is a growing body of research exploring syntactic capabilities of non-human animals. Much of this research focuses on the question of whether non-human animals can recognize languages with center embedding (e.g. $a^n b^m$), a product of context-free grammar, which is considered to be a proxy for the availability of hierarchical structure. It has been claimed that European starlings (Gentner et al., 2006), rhesus monkeys (Ferrigno et al., 2020), baboons (Malassis et al., 2020) and crows (Liao et al., 2022) are capable of recognizing hierarchical languages. Some have questioned the interpretation of this research on the grounds that non-human animals might be using alternative strategies for recognizing a string as being of the right kind, strategies that might not require these animals to represent the syntactic structure associated with the string (Beckers et al., 2012; Corballis, 2007). In any case, let us agree with the aforementioned research that nonhuman animals do indeed mentally represent context-free grammars. We wish to point out that the mere availability of such a grammar does not guarantee that these animals have the rich mental life that human grammars afford. As it turns out, System N is a context-free grammar given that conjunction is properly represented with the recursive rewrite rule $S \Rightarrow S \& S$, which is a context-free rule. However, this paper has shown that System N is not sufficient to generate concepts for things or events one has never experienced before, a striking attribute of human mind. Rather than asking just how complex the grammar of a given species is, we should ask how a given grammar rule or system transforms the cognitive systems with which it interacts. Indeed, this paper can be read as an attempt to highlight the fact that research on language evolution would benefit from focusing on the transformative cognitive consequences of the interface between syntax and thought systems rather than on syntax alone.
The paper has focused on the consequences of the possession of a grammar with Lambda Abstraction and quantification (i.e. System N+1) on human mental life. It is important to note that this is not to say that System N+1 exhausts all the complexity of human grammar but only that human grammar contains System N+1 (and likely more). For instance, we have focused on the consequences of abstraction over individual and event variables. However, natural language semantics also makes use of temporal variables, degree variables and place variables (which play a role in accounting for the entailment from “The cat jumped onto the mat” to “The cat was on the mat”, see Zwarts, 2017 for an overview). An interesting conjecture about different types of variables could be that they all originate within System N and that the capacity to abstract over them (with the rule of Lambda Abstraction) and quantify over such abstractions (with quantifiers) is uniquely human, whose cognitive consequences remain to be explored.

Secondly, a word must be said about quantification in System N+1. The way we handled quantifiers and quantification is not their standard treatment within formal semantics. A quantifier is typically further decomposed into a determiner (such as every, most, three, few ...) and its argument (every book, most students etc.). In a sentence like (49a), every is analyzed as a determiner whose first argument is the lambda term book (i.e. \( \lambda x.\text{is}(x, \text{BOOK}) \)). The merger of every and book, i.e. every book, is, then, a quantifier which syntactically maps the lambda term boring (i.e. \( \lambda x.\text{is}(x, \text{BORING}) \)) into a sentence with the meaning that the property of being boring is one of those properties that is true of each book. More intuitively, we can think of determiners as denoting relations between sets, between the sets characterized by their first and second argument. The determiner every can be said to denote the subsethood relation between its arguments as shown in (49b).

\[
\begin{align*}
(49) & \quad \text{a. Every book is boring.} \\
& \quad \text{b. every(book*, boring*)} \Leftrightarrow \{x: \text{is}(x, \text{BOOK})\} \subseteq \{x: \text{is}(x, \text{BORING})\}
\end{align*}
\]

This analysis of determiners has the merit of providing a representation of proportional determiners such as most, one-third, which can be shown to be impossible within standard first order logic (Barwise & Cooper, 1981). It is, however, known to over-generate determiner denotations. We can easily define a determiner, call it equi, which requires that each member of the determiner has the same cardinality (i.e. equi(A, B) \( \Leftrightarrow |A| = |B| \), see Keenan, 1996). Such a determiner is not known to exist in any natural language in which nominal determiners have been studied. This over-generation problem is typically handled by introducing constraints on determiner denotations. For instance, the determiner equi is eliminated from natural languages with a restriction known as the Conservativity Constraint (Barwise & Cooper, 1981; Keenan & Stavi, 1986), which, intuitively, says that the entities outside of the first argument of a determiner (book in (49a)) should be irrelevant to the truth conditions of the sentence involving the determiner (see von Fintel & Keenan, 2018 for a recent overview of constraints on determiner denotations).
this picture, Knowlton et al. (2021) claims that determiners should not be analyzed as relations between sets but as restricted quantifiers, where each “argument” has a distinct logical status and what we have called the first argument above behaves something like a topic for quantification. Under this analysis, the sentence in (49) has the following intuitive translation: “Relativized to the set of books, everything is boring”, which seems somewhat similar to the analysis of quantification within System N+1.

At this point, we may ask questions familiar from evolutionary theory: When did the transition from System N to System N+1 occur within the human lineage? What advantages did the possession of System N+1 confer to the individuals who had it so that the individuals endowed with this capacity ended up dominating the population? What this paper has to offer is a hypothesis about the formal statement of the what question (What is the grammar behind uniquely human concepts?), together with some suggestive notes on the brain functions involved in the implementation of Lambda Abstraction in the brain. Considering the significance of the basal ganglia (more generally the corticostriatal circuits involving the basal ganglia and the cortex) for procedural memory (Ullman, 2001b, 2004; Ullman et al., 1997; see Bolender et al., 2008, pp. 142–143 for a review of the neuroscience of procedural memory) and the role of the hippocampus in event conceptualization, ultimately a cognitive consequence of Lambda Abstraction, we find ourselves in agreement with the researchers who highlight the importance of the contributions of the subcortical brain regions to uniquely human linguistic capacities (Murphy et al. 2022; Shi & Zhang 2021).

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declarative memory, and that grammatical rules are processed by the procedural system.


