★ BRIEFS ★

Combinatorics for Metrical Feet

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1. Introduction

Halle & Vergnaud (1987) propose a convention on the parsing of elements into metrical feet — the Exhaustivity Condition — that requires all elements to belong to some foot, except for certain principled cases of extrametricality. However, the general consensus now prevailing is that even internal metrical elements can remain unparsed, failing to belong to any foot, generalizing the notion of extrametricality. Hayes (1995), Halle & Idsardi (1995), and Kager (1999), among many others, explicitly reject the Exhaustivity Condition. Hayes's comments are given in (1), Halle & Idsardi's are given in (2).

(1) "The upshot seems to be that in our present state of knowledge, it would be aprioristic to adhere firmly to a rigid principle of exhaustive prosodic parsing […]." (Hayes 1995: 110)

(2) "We also deviate from previous metrical theories by not requiring exhaustive parsing of the sequence of elements, that is we do not require that every element belong to some constituent […]." (Halle & Idsardi 1995: 440)

In this squib I will prove that the number of possible metrical parsings into feet under these assumptions for a string of n elements is $Fib(2n)$ where $Fib(n)$ is the n^{th} Fibonacci number.

2. Initial Observations

Disregarding prominence relations within the feet (that is, headedness), the possible footings for strings up to a length of three elements are shown in (3). Feet are indicated here by matching parentheses; elements not contained within parentheses are unfooted (that is, 'unparsed' in Optimality Theory terminology).

(3)
a. 1 element, 2 possible parsings: (x), x
b. 2 elements, 5 possible parsings: (xx), (x)(x), (x)x, x(x), xx
c. 3 elements, 13 possible parsings: (xxx), (xx)(x), (xx)x, (x)(xx), x(xx), (x)(x)(x), (x)(x)x, (x)x(x), x(x)(x), (x)xx, x(x)x, xx(x), xxx

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The number of possible footings is equal to every other member of the Fibonacci sequence, illustrated and defined as a recurrence relation in (4); see, for example, Cameron (1994).

(4) Fibonacci sequence: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, …
$Fib(0) = Fib(1) = 1$; for $n > 1$ $Fib(n) = Fib(n-1) + Fib(n-2)$

There is only one possible footing of a string of zero elements, so that it is also the case that the number of footings of zero elements is equal to $Fib(0)$.

3. Proof

Let $f(n)$ be the number of parsings of a string of n elements into metrical feet, not subject to the Exhaustivity Condition. We can derive a recurrence relation for the number of metrical feet in a string of length $n+1$ by dividing the string after the places where an initial foot could occur, as shown in (5).

(5)
a. no initial foot: x | …, n elements left, therefore $f(n)$ footings
b. 1–element foot: (x) | …, n elements left, therefore $f(n)$ footings
c. 2–element foot: (xx) | …, $n-1$ elements left, therefore $f(n-1)$ footings
d. 3–element foot: (xxx) | …, $n-2$ elements left, therefore $f(n-2)$ footings
…
e. n–element foot: (x…x) | , 0 elements left, therefore $f(0) = 1$ footing

Generally then, $f(n+1) = f(n) + \sum_{i=0}^{n} f(i)$

We then prove the general relation by induction on n. That is, we have shown by direct calculation that the relation holds for $n = 0$, 1, 2 and 3, (3), and now, assuming that $f(i) = Fib(2i)$ for i up to and including n, we will prove that $f(n+1) = Fib(2n+2)$. We begin with the recurrence relation derived in (5), pulling out the n^{th} term of the summation, shown in (6).

(6) $$f(n+1) = f(n) + f(n) + \sum_{i=0}^{n-1} f(i)$$

Substituting for $f(n)$ using the induction assumption gives (7).

(7) $$f(n+1) = Fib(2n) + Fib(2n) + \sum_{i=0}^{n-1} f(i)$$

Substituting for $Fib(2n)$ using the Fibonacci recurrence relation gives (8).

(8) $$f(n+1) = Fib(2n) + Fib(2n-1) + Fib(2n-2) + \sum_{i=0}^{n-1} f(i)$$

Substituting for *Fib(2n–2)* again using the induction assumption gives (9).

(9) $$f(n+1) = Fib(2n) + Fib(2n-1) + f(n-1) + \sum_{i=0}^{n-1} f(i)$$

Substituting for the last two terms using the *f(n)* recurrence relation gives (10).

(10) $$f(n+1) = Fib(2n) + Fib(2n-1) + f(n)$$

Substituting for *f(n)* again using the induction assumption gives (11).

(11) $$f(n+1) = Fib(2n) + Fib(2n-1) + Fib(2n)$$

Substituting the first two terms using the Fibonacci recurrence relation gives (12).

(12) $$f(n+1) = Fib(2n+1) + Fib(2n)$$

Again substituting using the Fibonacci recurrence relation gives (13), as required.

(13) $$f(n+1) = Fib(2n+2) \text{ Q.E.D.}$$

Having proved that if $f(n) = Fib(2n)$ then $f(n+1) = Fib(2n+2)$ for $n > 1$, and having $f(0) = Fib(0)$ and $f(1) = Fib(2)$, we have proved the relation for all non-negative n.

4. A Corollary

Given the above proof, substituting into the footing recurrence relation gives (14).

(14) $$f(n+1) = f(n) + \sum_{i=0}^{n} f(i)$$

$$Fib(2n+2) = Fib(2n) + \sum_{i=0}^{n} Fib(2i)$$

And, since from the Fibonacci recurrence relation we have $Fib(2n+2) = Fib(2n+1) + Fib(2n)$, therefore we derive (15).

(15) $$Fib(2n+1) = \sum_{i=0}^{n} Fib(2i)$$

That is, for example, $Fib(7) = Fib(6) + Fib(4) + Fib(2) + Fib(0) = 13 + 5 + 2 + 1 = 21$.

5. Conclusion

The number of non-exhaustive parsings of n elements into metrical feet (i.e. the number of non-exhaustive partitions of n elements) has been proven to be equal to $Fib(2n)$, the $2n^{th}$ Fibonacci number.

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