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A Question of Irresponsibility: Postal, Chomsky, and Gödel

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As part of a highly critical discussion of much recent work in linguistics, Paul Postal identifies "the most *irresponsible* passage written by a professional linguist in the entire history of linguistics" (Postal 2004: 296).[1] The brief of the present piece is to correct an error in Postal's reasoning. Space precludes a discussion of Postal's broader assault against what he perceives to be 'junk linguistics', although some brief remarks will be offered that pertain to the essence of his criticism.[2] The "most *irresponsible* passage" is from Noam Chomsky and includes the following remarks:

> [Expressions, i.e. the output of the language faculty — JC] are not entities with some ontological status; they are introduced to simplify talk about properties of [the language faculty — JC], and they can be eliminated in favor of internalist notions. One of the properties of Peano's axioms PA is that PA generates the proof P of "2 + 2 = 4" but not the proof P′ of "2 + 2 = 7" (in suitable notation). We can speak freely of the property "generable by PA", holding of P but not P′, and derivatively of lines of generable proofs (theorems) and the set of theorems without postulating any entities beyond PA and its properties. (Chomsky 2001: 41f.)

Chomsky's general point in this passage, I take it, is that the empirical coverage of any theoretical discourse can be rendered as a commitment to a set of the relevant entities (e.g., the set of possible trajectories of Halley's Comet, the set of possible electrons, the set of possible tigers, or, indeed, the set of expressions as the idealized output of the language faculty). Such ontological commitment to the sets of the relevant entities, however, is not required for the explanatory goals of the given sciences, unless, of course, the science is a branch of mathematics that is concerned with large sets and their properties, and there the identity of the entities is irrelevant. In linguistics, at least, "[n]o 'Platonism' is introduced, and no 'E-linguistic' notions: only biological entities and their properties" (Chomsky 2001: 42).

Now, of course, no scientific theory is merely concerned with what finitely happens to obtain; for example, zoology is concerned with tigerhood, as it were,

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1 For, to my mind, a sound review of Postal's more general remarks against 'junk linguistics', see Boeckx (2006).

2 For wider discussion of the same themes, see Collins (forthcoming).

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not its contingent instantiation. Every theory, we may say, has an infinite import. This is because the very notion of explanation is modal insofar as it must support counterfactuals. Thus, a law does not describe phenomena but tells us what will occur under any conditions that satisfy the properties the theory posits. For example, Newton's laws don't purport to *describe* our solar system (unlike Kepler's 'laws'), but instead tell us what will occur in any circumstances that are covered by the concepts of classical mass and force, which our solar system happens to realize (within certain parameters — forget about twentieth century developments). In this sense, Newton's laws tell us about infinitely many possible systems, even though our universe is finite (we presume).[3] The same holds in the case of linguistics. A formal theory tells us about infinitely many possible states the human mind/brain can fall into, without committing itself to the idea that the mind/brain is infinite, or, of course, that there are infinitely many sentences anywhere at all, not even in Plato's heaven. To be sure, we need to employ the notion of an infinity of expressions, in Chomsky's sense, much as we are required to think about infinitely many states of any physical system (theorized, say, in terms of Lagrangians or Hamiltonians). My present point is merely that such notions, while essential in the modal sense explained above, don't attract our ontological commitment, at least not if we are working within the theory (cf. Feferman's 1998 position on the relation between science and mathematics).[4] If all this is so, linguistics looks to be in the same boat as any other science. Let us now turn to the detail of Chomsky's remarks quoted above, which Postal finds so objectionable.

Postal thinks Chomsky's reasoning is particularly irresponsible because Chomsky distorts the mathematical case (PA, a formalized theory of elementary arithmetic) to lend weight to his ontological claims about linguistics. The average linguist, we may presume, is not familiar with the relevant mathematics and so is liable to be taken in by Chomsky's conceit.

Postal reads Chomsky as suggesting that the standard metatheoretic results for a theory of elementary arithmetic (PA) do not require the postulation of a set of truths. Such a suggestion, however, *apparently* contradicts Gödel's incompleteness theorems; indeed, the results would not even be formulatable: "[To show that] a system like PA is complete, one must consider the relation between two sets […] the set of theorems of [PA] and the set of truths about [the natural numbers]", both infinite (Postal 2004: 303). So, if Chomsky's remarks are correct "not only would it be impossible to *prove* Gödel's incompleteness theorems, it would be impossible to even *formulate* them" (Postal 2004: 303). Postal's reasoning is awry.

First, Postal appears to have been misled by the familiar informal

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3 I don't presume that 'laws' necessarily determine everything that happens to occur, for many events or states might fall outside of the scope of the concepts that enter into general laws. For example, presumably, no law of physics will tell us the number of the inner planets. See van Fraassen (1989) for, to my mind, an excellent discussion of these issues.

4 Controversy on this issue appears to arise from the common talk in linguistics and philosophy of natural languages being infinite. The infinity of English, as it might be, however, is not a phenomenon. The phenomenon is that particular organisms are continuously novel in their speech and understanding, which we theorize in terms of unbounded generation.

presentation of Gödel's (first) incompleteness theorem as showing that there are unprovable truths of PA. There is, I should say, nothing wrong with such a characterization just so long as one does not take it to reflect the actual mechanics of Gödel's proof (there are many ways of presenting Gödel's theorems; here we are concerned with Postal's claims about what is *essential* to them). Gödel set himself the task of showing that finitary PA is incomplete on its own terms, i.e., without making any non-finitary assumptions, such as the set of every PA truth, for such a set is inadmissible in finitary metamathematics. Gödel's result is wholly syntactic/combinatorial and so finitary in the sense that it can be carried out within PA itself.[5] Gödel wrote:

> The method of proof […] can be applied to any formal system that, first, when interpreted as representing a system of notions and propositions, has at its disposal sufficient means of expression to define [such] notions […] and in which, second, every provable formula is true in the interpretation considered. The purpose of carrying out […] the proof with full precision in what follows is, among other things, to replace the second of the assumptions just mentioned by a purely formal and much weaker one.
> (Gödel 1931/1986: 151; cf. p.181)

As mentioned above, Gödel adopts this weaker formulation in order to be consistent with Hilbert's finitary scruples on metamathematics. The weaker formulation substitutes formal (syntactic/combinatorial) consistency principles — 'simple consistency' and 'ω-consistency' — for 'truth'. This syntactic character of the result became clearer still with Rosser's later technique of generating undecidable formulae without appeal to ω-consistency.

So, Postal's claim that the relevant theorems are unformulatable without the positing of a set of truths is false; the opposite is the case, at least if the theorem is not to be question begging against finitary metamathematics of the kind that concerned Gödel.[6]

Secondly, the general notions of completeness and incompleteness do not require the postulation of two sets; both notions can be construed in finitary proof-theoretic terms, which are the notions Gödel employed. Gödel's proof proceeds by a lemma that allows for the construction of PA formulae that are such that, if PA is consistent, then neither the formulae nor their negations are theorems of PA ('simple consistency' also needs no postulation of a set of truths; PA is consistent if and only if there is at least one PA formula that is not a PA theorem). The incompleteness of PA follows, on the assumption that it is consistent (the 'undecidable' formulae become theorems in a system richer than

5 The so-called 'Hilbert Program' was dedicated to establishing finitary consistency proofs for classical mathematics (arithmetic, analysis, etc.), proofs that, *inter alia*, make no essential appeal to 'actual infinities' by way of, for example, unrestricted use of excluded middle or negations of universal quantifications. There are currently many different ways of construing finitary scruples. Postal (2004: 301) accuses of Chomsky of "distortion" for conflating the axioms of PA with a proof theory. It is perfectly standard, however, to use 'PA' to designate a first-order formalization of the axioms along with a given proof theory, as Chomsky makes clear in his talk of "suitable notation".

6 It is worth noting that Gödel's completeness proof for first-order logic *does* require the postulation of a set of truths in that the proof shows that there is *no* first-order satisfiable formula that is not a theorem.

PA, which is what the theorem amounts to, in one sense). So, one can prove Gödel's theorem, as Gödel himself did, without the postulation of a set of truths.[7]

Postal is correct to think that Gödel refuted the ambitions of strict finitary proof theory as laid out by Hilbert, but Chomsky does not even hint at the contrary.[8] Chomsky's point is almost banal. One can, by familiar techniques, codify first-order PA as a Turing machine program (in fact, the main business of Gödel's 1931 paper was to show that the relevant metamathematical concepts are all decidable apart from 'provable', which is only semi-decidable). The program is a finite object that has an infinite output. One is not thereby committed to the output being a set in Plato's heaven anymore than an astronomer is committed to the infinite set of the trajectories of Halley's Comet. In formal proof theory, one thinks of a proof as a (finite) set of formulae drawn from the relevant formal language. Whether a formula is a theorem ("2 + 2 = 4") or not ("2 + 2 = 7") is determined by the axioms and rules of inference of the theory (the properties of the 'finite object'). Imagine coming a across a machine, like Paley and his watch, that spews out arithmetical formulae. One can investigate the 'program' of the machine as a finite object realized in the physical structure of the machine. One may, for instance, wonder whether there is a formula compatible with the 'program' (well-formed), which is such that neither it nor its negation will be produced. Such an approach towards the machine does not involve "postulating any entities beyond [the program] and its properties". Of course, Gödel showed that any formal system of sufficient richness (capable of representing arithmetic) will express undecidable propositions, but this result does not flow from looking beyond the finite system (incompleteness follows from finitary reasoning, which was Gödel's point), nor does it show that any particular metaphysics of mathematics is correct.[9] Likewise, linguistics may freely posit a finite object realized in the human mind/brain and theorize over its derivations and sets of expressions without thereby committing itself to any entities beyond the biology of the mind/brain. If there are such entities, then they have to be established, much as, *mutatis mutandis*, Gödel established the limitations of certain formalizations, but the mere availability of a set theoretical formulation does not take one beyond whatever entities are one's primary concern.

Postal raises serious issues about the interpretation of formalism vis-à-vis phenomena, or, equivalently, the status of abstractions in science. My brief has been to argue that no quandaries over this issue selectively strike at work in linguistics; more specifically, the status and interpretation of Gödel's results have

7 Via techniques introduced by Tarski, one can define a truth predicate for the language of PA in a richer metalanguage, where the undecidable formulae generable in PA are rendered as theorems, i.e. true. It is in this sense that the (first) incompleteness theorem shows that there are unprovable truths of PA (keeping to the resources of PA itself). The class of second-order PA truths, however, remains undecidable.

8 One can be finitary, however, without being Hilbertian. Kleene (1986: 138) notes: "Gödel's second incompleteness theorem, rather than ending efforts at finding finitary consistency proofs, pointed out the road to success". That is, Gödel showed that certain *elementary* procedures were inadequate.

9 Gödel was a Platonist by inclination, but he did not take his incompleteness results to establish the truth of Platonism; indeed, he was also sympathetic to Leibniz and Kant on the philosophy of mathematics. See Gödel (1961/1995).

no particular bearing on linguistics. Postal's remarks are certainly not the most irresponsible in the history of linguistics; equally, to say the least, they go no way to show that Chomsky's remarks are either.

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