Wittgenstein and Biolinguistics: Building upon the Second Picture Theory

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Circa 1930, Wittgenstein began to develop a theory of semantics in terms of distinct representational systems (calculi) each constructed from measurement scales. Impressed by the heterogeneity of measurement scaling, he eventually abandoned the effort. However, such a project can be continued in the light of later developments in measurement theory. Any remaining heterogeneity can be accounted for, plausibly enough, in terms of the facultative nature of the mind/brain. Developing such a theory is potentially a contribution to biolinguistics. The symmetries and asymmetries of the measurement scales suggest self-organization in brain activity, further suggesting a connection between such a neo-Wittgensteinian approach to the thought systems and minimalist approaches to syntax.

Keywords: calculus model; conceptual-intentional systems; measurement theory; *Satzsysteme*; picture theory; self-organization

1. Introduction

Practitioners of biolinguistics, the attempt to understand language by situating it within a biological framework, are often drawn to appeals to symmetry and beauty (Chomsky 2005). Lyle Jenkins (2000: 147) suggests that an important strategy in biolinguistics should be to seek confirmation of Einstein's observation that "symmetry dictates design". Juan Uriagereka (1998) notes that the branching form of a structural description exhibits dilation symmetry, by virtue of being fractal. The point is to discover how much of syntax 'comes for free', in the sense of being directly the result of physics, as opposed to the molding and shaping of differential reproduction as genetically encoded.

> In the study of the inorganic world, for mysterious reasons, it has been a valuable heuristic to assume that things are very elegant and beautiful. If physicists run across a number like 7, they may assume that they have missed something, because 7 is too ridiculous a number: it must really be 2^3, or something like that. […] Similar intuitions have been reasonably successful in the study of language. If they are on target, it may mean that language is rather special and unique, or that we do not understand enough about other organic systems to see that they are much the same, in their basic structure and organization. (Chomsky 1996: 30)

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Despite occasional inclusive remarks, such as the above reference to "other organic systems", this approach has usually focused on the computational core of the language faculty and its interfaces. In the biolinguistics literature, it is more common to find observations regarding beauty limited to syntax, such as the following: "[T]he principles of language are determined by efficient computation and language keeps to the simplest recursive operation designed to satisfy interface conditions in accord with independent principles of efficient computation. In this sense, language is something like a snowflake, assuming its particular form by virtue of laws of nature" (Berwick & Chomsky 2016: 71). Less attention has been paid to the conceptual-intentional systems, the systems of thought.

One aim of this article is to show that the picture theory of Ludwig Wittgenstein (ca. 1930) holds promise as a potentially biolinguistic approach to the thought systems, by reason of revealing symmetry and beauty in those systems. When the picture theory of 1930 is developed as a generative system—contrary to Wittgenstein's own wishes, it must be said—then the relevance of third-factor considerations (Chomsky 2005) to the thought systems becomes apparent. The systems of thought also turn out to be like a snowflake, in other words.

2. The Second (Version of the) Picture Theory

Wittgenstein's picture theory of meaning has a long and interesting history, even if not an entirely happy one. An atomistic version of the theory forms the centerpiece of his first book, *Tractatus Logico-Philosophicus* (Wittgenstein 1922), later replaced by a holistic version in his second book *Philosophical Remarks*, completed in 1930 (Wittgenstein 1975a). While the notion of a measurement scale was important for the theory from the very beginning,[1] the concept of measurement scaling assumes a much greater and more explicit role in the later holistic version. It is the role of scaling which, as I shall argue, reveals how beauty and symmetry enter into processes in the thought systems.

Unfortunately, Wittgenstein was developing this approach to semantics primarily in the years 1929 and 1930, prior to important developments in measurement theory. As a result, the theory was never fully developed. In fact, his frustration in regard to the hope of fully developing the theory eventually led to his adopting an anti-theoretical stance, most famously elaborated upon in *Philosophical Investigations* (Wittgenstein 2001). However, when one considers advances in measurement theory, one becomes optimistic of further developing the holistic version of the picture theory. In fact, it is reasonable to expect the

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1 In his earliest reference to the picture theory, Wittgenstein mentions measurement and picturing as though they are essentially the same relation between proposition and fact. On 24 November 1914, he wrote in his notebooks that "[p]roposition and situation are related to one another like the yardstick and the length to be measured. […] In the proposition we hold a proto-picture up against reality" (Wittgenstein 1979: 32). Also note "[t]he proposition is a measure of the world" (Wittgenstein 1979: 41), written in the following March (cf. Wittgenstein 1922: §2.1512). For a later perspective on this early appeal to scaling, see (McGuinness 1979: 185, Stern et al. 2016: 2332–2376).

development of a kind of generative grammar for the representational systems which Wittgenstein posited in his second book, i.e. a grammar for specifying calculi, just as the particular grammar of a natural language specifies structural descriptions. This would not, however, be a replacement for the Merge-based generative grammar familiar in minimalist linguistics, since it would pertain to systems outside the narrow language faculty. It would be complementary.

The version of the picture theory at issue here is the holistic version circa 1930, formulated nearly a decade after *Tractatus*. One could even, properly enough, speak of the holistic version as a *second* picture theory. The term 'picture theory', in fact, is so closely associated with *Tractatus*, it may be wise to avoid the phrase 'picture theory' in reference to the later holistic view. From now on, I will speak of 'Wittgenstein's Second Theory' or, simply, 'The Second Theory', thus minimizing use of the word 'picture'. The Second Theory is a partially developed viewpoint, found primarily in Wittgenstein's "Some Remarks on Logical Form" (Wittgenstein 1929), *Philosophical Remarks* (Wittgenstein 1975a), and conversations recorded by Friedrich Waismann (McGuinness 1979). (I will, even so, sometimes cite pre-1929 work as belonging to The Second Theory when it seems appropriate to do so, on the plausible assumption that many ideas from the first picture theory were meant to carry over into The Second Theory.)

The core idea in The Second Theory is that semantics is measurement: The mind/brain uses measurement scales to make pictures of 'spaces', where a space is a range of logical possibilities for some domain of phenomena (Wittgenstein 1975a: Ch. IV & p. 317). The semantic properties of a scale are partially a matter of the internal structure of the scale, its 'multiplicity', and also partly a matter of how the scale is used (Wittgenstein 1922: §3.326). The use crucially depends upon the multiplicity; without the right multiplicity, there could be no relevant use. Scales are sometimes combined into complex structures, 'calculi' (Wittgenstein 1975a, McGuiness 1979), each scale of a given calculus being a picture of a dimension of the corresponding space. A given calculus is thus a picture of the pertinent space. (It is the *construction* of these calculi which raises the possibility of a generative grammar of sorts, a point to be expanded upon later.)

As an example of holistic picturing, consider the waggle dance of the honeybee. A bee observing another bee perform the dance forms a picture in its brain homomorphic to the dance. This picture presupposes a multi-dimensional space, each dimension of which corresponds to a scale of measurement. The space, as represented in the brain, is a picture of the corresponding space in the external world, the latter being a range of possible locations of food or water. The specific dance is a picture of a possible fact, just as the space of possible dances (i.e., the calculus) is a picture of the space of all possible facts about location. The meaning of the mental representation crucially depends upon the internal structure of the calculus, but also depends upon the use made by the bees, the latter crucially depending upon the former. Semantics is holistic in that the specific dance has no meaning unless the calculus has meaning, i.e. the former is only a picture of a fact by reason of the latter being a picture of the pertinent space of possible facts.

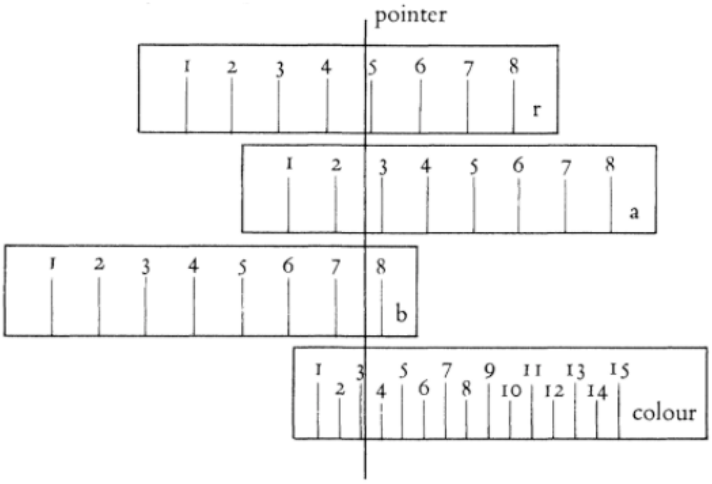

Figure 1: Representation of a calculus, or Satzsystem *(propositional system). This calculus pictures the ranges of possible positions, colors, and radii of a circle in a two-dimensional space. The example is from Wittgenstein (1975a: §84).*

Writing to G. E. Moore in 1930, Russell noted, in reflecting on recent conversations with Wittgenstein regarding his manuscript *Philosophical Remarks*, that Wittgenstein "uses the words 'space' and 'grammar' in peculiar senses, which are more or less connected to each other. He holds that if it is significant to say 'This is red,' it cannot be significant to say 'This is loud.' There is one 'space' of colours and another 'space' of sounds. […] Mistakes of grammar result from confusing 'spaces'" (Russell 1951: 297). The grammar of a given calculus consists of the measurement scales making up its dimensions. "The syntax of ordinary language […] does not in all cases prevent the construction of nonsensical pseudo-propositions (constructions such as 'red is higher than green' […])" (Wittgenstein 1929: 162). "Red is higher than green" violates grammar because the scales which constitute the color calculus are distinct from the scales constituting the pitch calculus. The sentence "This is red and loud", when it expresses a proposition, must thus be analyzable, on the level of the thought systems, into something along the lines of "A is red, and B is loud". Each calculus, loosely speaking, represents its own world; the logical connectives (such as 'and', 'or', etc.) link these worlds.

Note, however, that this is only a *theory* of mental representation if measurement scaling is a specific type of thing. Wittgenstein was sensitive to the various sorts of measurement scaling, a point to be expanded upon later. Struck by this variety, he eventually came around to the view that logic and language have no essence. There is too much variety for theory to be possible, or so he came to believe. Note Wittgenstein's eventual anti-essentialism with regard to both number and language in *Philosophical Investigations* (Wittgenstein 2001: §§65–71). Given his earlier meditations on the varieties of scaling (Wittgenstein 1975a, McGuinness 1979) and the fact that the various scales can be used to define different types of number (Ellis 1966: Ch. IV, Wiese 2003: Ch. 1), an anti-essentialism with regard to scaling was almost certainly on his mind in writing about number in *Investigations*.

However, as would be revealed by advances in measurement theory beginning in the late 1940s, there is some degree of beauty and formal simplicity underlying the types of measurement scales. It is at this point that the prospect of self-organization in neural activity arises with regard to the Second Theory. I suggest that, encouraged by these developments, one should pick up the thread and continue theorizing precisely where Wittgenstein left off. If the Wittgensteinian calculi provide insight into the representational powers of non-linguistic faculties, then the beauties of measurement theory could reveal that language is not so unique in this regard, since other systems also exhibit beauty.

There is another source of heterogeneity among logical forms, namely the contrast between forms arising within a calculus versus the recursive/combinatorial operation linking representations to yield molecular forms. In other words, there is the distinction between the language faculty and the thought systems, especially in terms of how the latter function apart from the influence of language. Advances in measurement theory will not make this kind of heterogeneity go away. However, biolinguists should be comfortable with it, as it merely reflects the facultative nature of mind (Hauser et al. 2002).

3. The Atomic and the Molecular

The Second Theory, like the earlier version of the picture theory in *Tractatus*, assumes a potential analysis of representations into atomic representations. Each calculus determines logical relations among the atomic representations belonging to it.

> If we try to analyze any given propositions we shall find in general that they are logical sums, products or other truthfunctions of simpler propositions. But our analysis, if carried far enough, must come to the point where it reaches propositional forms which are not themselves composed of simpler propositional forms. We must eventually reach the ultimate connection of the terms, the immediate connection which cannot be broken without destroying the propositional form as such. The propositions which represent this ultimate connexion of terms I call, after B. Russell, atomic propositions. They, then, are the kernels of every proposition, *they* contain the material, and all the rest is only a development of this material.
>
> (Wittgenstein 1929: 162–163)

To say that P is an atomic proposition—what Wittgenstein usually referred to as an elementary proposition—is to say that, if P were written as a compound proposition, its compound form would not be essential to its truth-value. For example, if P is "a is red" one could rewrite it as "a is red, and if b is blue then b is blue". This would be extensionally equivalent to the original proposition. Given that "if b is blue then b is blue" is logically true, it can be deleted from the proposition without altering the truth conditions (see Wittgenstein & Waismann 2003: 245). This stands in contrast to "a is red, and b is blue" where a and b are distinct space-time regions. This latter would be a molecular proposition, since one cannot remove either conjunct without arriving at a proposition with different truth conditions than the original. The elementary propositions can be combined, according to a recursive operation, to form molecular propositions (Wittgenstein 1922: §§6–6.01). For example, if P, Q, and R are elementary propositions, then P & (Q & R) is a molecular proposition. Connectives introduce new logical relations, including those between calculi.

In *Tractatus*, Wittgenstein affirmed that there are no logical relations between elementary propositions other than consistency and self-implication, which are merely degenerate or limiting cases. True logical relations arise at the molecular level, and are captured in truth tables, according to this early view. However, by 1929, Wittgenstein came to realize that there are logical relations of exclusion among the readings of a single measurement scale even without molecular structure. In any specific case of measurement, a given reading logically excludes every other reading for that scale: "But from 'a is now red', 'a is now not green' follows, and so elementary propositions in this sense aren't independent of each other like the elementary propositions in the calculus I described earlier—a calculus to which I assumed the entire use of propositions must be reducible—seduced by a false concept of such a reduction" (Wittgenstein 2013: 82). This is a general property of measurement: "[A] point mass can only have *one* velocity at a time, there can only be *one* charge at a point of an electrical field, at *one* point of a warm surface only one temperature at one time, at one point in the boiler only *one* pressure etc." (Wittgenstein 1975a: §81).

Furthermore, there are arithmetical relations among the readings of a scale, e.g. the transitivity of an ordinal scale, which cannot be captured by truth tables. In fact, it is by reason of these arithmetical relations that the word 'calculus' is appropriate. Wittgenstein thus concluded that "[o]ne could surely replace the logic of tautologies by a logic of equations" (quoted in Hintikka 1996: 85). This is an overstatement, since one would also have to include relations of greater-than and less-than, e.g. in ordinal scaling. It is also an overstatement by reason of the fact that one would still need a logic of tautologies at the molecular level. Even so, the quote is useful in illustrating Wittgenstein's perception that arithmetical relations are even more basic than truth-functional relations. Truth-functional relations presuppose these calculi, thus turning on its head the older view (Russell 1919) that arithmetic was reducible to logic, or some combination of logic and set theory.

4. Prelinguistic Systems

The picture theory of Wittgenstein, both in *Tractatus* and circa 1930, was intended as an account of mental representation. This is explicit in *Tractatus*: "The logical picture of the facts is the thought" (Wittgenstein 1922: §3), and "[t]he thought is the significant proposition" (Wittgenstein 1922: §4).[2] Note also the remark Witt-

[2] This is why Steven Pinker was unfair to Wittgenstein in speaking scornfully of his remark "*The limits of my language* mean the limits of my world" (Wittgenstein 1922: §5.6; cf. Pinker 2007: 134). At any rate, Pinker was unfair in trying to link the remark to the Sapir–Whorf hypothesis. Wittgenstein was not speaking of the limits of German or any other natural language, at least not specifically. He was, rather, speaking of the sum total of all possible mental representation. Perhaps one could still question the statement, or puzzle over what

genstein made in a lecture of 1930, as recorded in G. E. Moore's notes: "What sort of harmony must there be between thoughts & the world? Only that the thought must have logical form; & without this it wouldn't be a thought" (quoted in Stern et al. 2016: 2520). The point is that the calculus must have the same multiplicity as the relevant space of facts, the calculus here being something mental.

Given that the picture theory, including Wittgenstein's Second Theory, is a theory of mental representation, it is in no way alarming to realize that some of Wittgenstein's calculi are not language-like. This simply reflects the fact that not all thought is language-like. Some Wittgensteinian calculi are not language-like by virtue of not exhibiting discrete infinity (Bolender 2017). Within the biolinguistic paradigm, discrete infinity is understood to be biologically rare and, within the domain of cognition, uniquely human and wholly due to language (Hauser et al. 2002, Berwick & Chomsky 2016). Specifically, it is taken to be the result of recursion, usually understood as Merge (Chomsky 1995). Hence, biolinguists seldom recognize any reason to posit a Merge operation in the systems of thought.[3,4]

Consider the calculus illustrated in Figure 1 above. It exhibits a kind of infinity but not the infinity resulting from the recursive combination of discrete constituents. It exhibits infinity of the same sort as the waggle dance of the honeybee: "Between any two signals there is in principle another, signaling a distance in between the first two, and this continues down to the ability to discriminate" (Chomsky 1988: 169). One finds here, in other words, either discrete finitude (finite due to performance limitations) or continuum infinity.

exactly it was supposed to mean, but it is not as patently absurd as saying that the limits of German were the limits of Wittgenstein's world.

3 A reviewer for this journal suggests that the claim made here conflicts with the following passage from Chomsky: "[T]he Basic Property is generation of an unbounded array of hierarchically structured expressions mapping to the conceptual-intentional interface, providing a kind of 'language of thought'—and quite possibly the only such LOT, though interesting questions arise here" (Chomsky 2016: 13). The referee suggests that this passage indicates that Chomsky believes that the thought systems also utilize Merge.

My interpretation of the passage differs, however. In fact, the whole point in Chomsky's speaking of interfaces is to imply that the systems of thought do not produce indefinitely many hierarchically structured expressions independently of language. Chomsky speaks of the language faculty as LOT, because he recognizes it as playing a role in cognition by means of the CI interface. (For a brief discussion of one of the "interesting questions" which Chomsky alludes to, see fn. 4 right below.) That Chomsky does not equate language with thought itself should also be clear from the following quote from the same book: "[F]undamental properties of language design indicate that a rich tradition is correct in regarding language as essentially an instrument of thought, *even if we do not go as far as Humboldt in identifying the two*" (Chomsky 2016: 16, emphasis added).

4 One apparent anomaly for this approach is the fact that people who lose syntactic ability in adulthood sometimes continue to show evidence of recursion in cognition, for example in mathematics. But this does not show that recursion is an intrinsic feature of the thought systems, since "language grammar might provide a 'bootstrapping' template to facilitate the use of other hierarchical and generative systems, such as mathematics. However, once these resources are in place, mathematics can be sustained without the grammatical and lexical resources of the language faculty" (Varley et al. 2005: 3523). Even if the thought systems do somehow form something analogous to the Merge operation, they may only be able to do so using language as a kind of ladder which is subsequently discarded at some point in ontogenesis. Its being discarded would be evident in cases of agrammatism when accompanied by numerical literacy.

That at least some calculi exhibit continuum infinity is clear from Wittgenstein's first published attempt to articulate The Second Theory, or a fragment thereof.

> If, now, we try to get at an actual analysis, we find logical forms which have very little similarity with the norms of ordinary language. We meet with the forms of space and time with the whole manifold of spacial and temporal objects, as colours, sounds, etc., etc., with their gradations, continuous transitions, and combinations in various proportions, all of which we cannot seize by our ordinary means of expression. And here I wish to make my first definite remark on the logical analysis of actual phenomena: it is this, that for their representation numbers (rational and irrational) must enter into the structure of the atomic propositions themselves. I will illustrate this by an example. Imagine a system of rectangular axes, as it were, cross wires, drawn in our field of vision and an arbitrary scale fixed. It is clear that we then can describe the shape and position of every patch of color in our visual field by means of statements of numbers which have their significance relative to the system of co-ordinates and the unit chosen. Again, it is clear that this description will have the right logical multiplicity, and that the description which has a smaller multiplicity will not do.
> (Wittgenstein 1929: 165)

The calculus, in this case, must have representational powers corresponding to the real numbers in order to be homomorphic with the space of possible visual sensations. It is in this sense that the atomic propositions have real numbers coded into their structure. Relations of logical implication among numbers turn out to be primary, since they can be found at the level of the atomic proposition.

The reference to numbers here does not mean that the calculus in question is a system of discrete infinity. For a Wittgensteinian calculus is not, in every case, a system for combining discrete objects. The notion of calculation or equation, at least as it appears in early intermediate Wittgenstein, should not be taken to involve discrete objects or, at least, not in every case. In some cases, a number is not a digit but a point in a continuous space. Consider Wittgenstein's attempt to illustrate a calculus for representing the space of possible colors (Figure 2); the system represented is not a particulate system but a blending system.[5] The calculus in Figure 2 only contains numbers insofar as each point in the continuous structure represents a number.[6]

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5 Abler (2005) observes that complex systems divide into two types: particulate and blending. A particulate system consists of building blocks in hierarchical arrangement. Such a system is combinatorial and open-ended, language being a classic illustration. A particulate system stands in contrast to a blending system.

> Continuous variation along one or a few dimensions is the very definition of a blending system. In spite of the panoramic grandeur and local fascination, geology is a blending system. Hills, plains and valleys don't form combinations with one another to create something with properties beyond those of hills, plains, and valleys. Not the way atoms do." (Abler 2005: 70–71)

Wittgenstein's calculus for representing possible colors is a blending system, as it exemplifies continuous variation along dimensions.

6 Wittgenstein would later reject the double-cone representation appearing in Figure 2 (Wittgenstein 1977). But he rejected it for not including enough dimensions (e.g. luminosity, transparency), leaving the current point at issue unaffected.

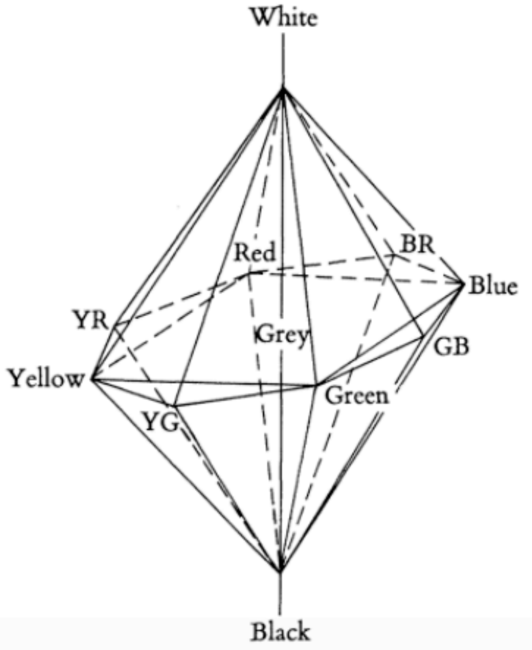

Figure 2: Wittgenstein's double-cone representation of color space. "If we represent the colours by means of a double-cone, instead of an octahedron, there is only one between on the colour circle, and red appears on it between blue-red and orange in the same sense as that in which blue-red lies between blue and red. And if in fact that is all there is to be said, then a representation by means of a double-cone is adequate, or at least one using a double eight-sided pyramid is" (Wittgenstein 1975a: §221).

Figure 2 illustrates a continuous calculus, not Merge-like in any way. It is true that the real numbers are definable in terms of operations on the naturals, and the naturals exhibit discrete infinity. However, in Figure 2, one finds a system of infinite multiplicity not presupposing discrete infinity. One has here the reals without the naturals.

I earlier noted that *some* Wittgensteinian calculi are not language-like by reason of exhibiting continuum infinity; note that there are others which are not language-like by reason of exhibiting discrete finitude. Once again, this is an encouraging feature of Wittgenstein's Second Theory since it underscores the potential here for understanding the form of representations outside the language faculty. Before considering examples from Wittgenstein, let's review the distinctions in question:

> To put it simply, each sentence has a fixed number of words: one, two, three, forty-seven, ninety-three, etc. And there is no limit in principle to how many words the sentence may contain. Other systems known in the animal world are quite different. Thus the system of ape calls is finite; there are a fixed number, say, forty. The so-called bee language, on the other hand, is infinite, but it is not discrete. A bee signals the distance of a flower from the hive by some sort of motion; the greater the distance, the more the motion. Between any two signals there is in principle another, signaling a distance in between the first two, and this continues down to the ability to discriminate.
>
> (Chomsky 1988: 169)

In addition to those Wittgensteinian calculi which are formally analogous to bee signals, there are also such systems which are formally analogous to ape calls. In a conversation with Friedrich Waismann early in 1930, Wittgenstein observed that "[w]hat I admittedly do not know is how large the domain of arguments is. And there might, for example, be only two. (Telephone dialling: free, in use—here we know that only these two values exist and they depict reality. An intermediate position does not signify anything. No transition.)" (quoted in McGuinness 1979: 90). In other words, a given dimension of such a system need not be continuous, but may consist of a finite set of logical possibilities. Wittgenstein also made the point in writing.

> What we have recognized is simply that we are dealing with yardsticks, and not in some fashion with isolated graduation marks. […] We might think of the signals on a ship: 'Stop', 'Full Speed Ahead', etc. Incidentally, they don't have to be yardsticks. For you can't call a dial with two signals a yardstick. […] If I say I did not dream last night, I must still know where I would have to look for a dream (i.e. the proposition 'I dream', applied to this situation can at most be false, it cannot be a nonsense). I express the present situation by a setting—the negative one—of the signal dial 'dreams—no dreams'.
> (Wittgenstein 1975a: §§84 & 86)

The examples which Wittgenstein gives here are recognizable as instances of what would later be called 'nominal scaling' (Stevens 1946). A nominal scale consists of labels, each label corresponding to a property. On a questionnaire, for example, one might encounter the question "Are you a smoker? Yes or no". Or one might encounter "State your political party". Each of these questions measures the thing, such as a person, simply by putting it into the proper slot in a list of categories. Hence, among the Wittgensteinian calculi, one finds both alternatives to discrete infinity, namely continuum infinity and discrete finitude.

In his *Tractatus*, Wittgenstein may have succeeded in avoiding the appearance of heterogeneity by naively assuming that all measurement is reducible to nominal scaling. Consider the following illustration of representation in *Tractatus*:

> An illustration to explain the concept of truth. A black spot on white paper; the form of the spot can be described by saying of each point of the plane whether it is white or black. To the fact that a point is black corresponds a positive fact; to the fact that a point is white (not black), a negative fact. If I indicate a point of the plane […], this corresponds to the assumption proposed for judgment, etc. etc. (Wittgenstein 1922: §4.063)

The only possible judgment for a point, in this case, would be whether it belongs to the scale (being black) or does not (being "white", apparently a catch-all for simply not being black)—a very simple form of nominal scaling. This sort of picture theory would be atomistic, but falls short of plausibility by overlooking phenomena which resist nominal scaling. Nevertheless, it still serves to illustrate how a calculus can be composed of nominal scales. This type of calculus is essentially identical to what Quine would later call a "matrix of alternatives":

> The notion of information is thus clear enough nowadays when properly relativized. It is central to the theory of communication. It makes sense relative to one or another preassigned matrix of alternatives—one or another checklist. You have to say in advance what features are going to count. Thus consider the familiar halftone method of photographic illustration. There is a screen, say six by six inches, containing a square array of regularly spaced positions, say a hundred to the inch in rows and columns. A halftone picture is completely determined by settling which of these 360,000 points are black. Relative to this screen as the matrix of alternatives, information consists in saying which places are black. Two paintings give the same information, relative to this matrix, when they determine the same points as black.
>
> (Quine 1986: 4)

This example illustrates how a calculus exhibiting discrete finitude, even when based upon the simplest kind of nominal scaling, can be informationally rich.

5. Scales as Constituting Essence

Wittgenstein's abandoned second philosophy of logic is a torso in the same sense that one speaks of an unfinished work of art as a torso. One can make progress in developing the torso by considering developments in measurement theory. The aim here is to arrive at a conception of logical form for *natural logic*, by which I mean the theory of how logical relations are represented in the mind/brain. Given that the logic in question is natural logic, a more developed form of Wittgenstein's second philosophy of logic has the potential to connect with work in cognitive science.

Measurement scales also play a fundamental role in Relational Models Theory (RMT) in cognitive anthropology (Fiske 1990, 1991, 1992, 2004a). In contrast to Wittgenstein's eventual theoretical nihilism, however, RMT is usually understood to be an attempt at formulating a generative grammar for social relations. Reflecting on why RMT is interpreted so differently from Wittgenstein's philosophy of logic, despite both being similar, even in very fundamental ways, will throw light on how the Second Theory can be understood in generative terms. RMT not only provides a model for theoretical development, it is an actual example of the application of calculi to spaces of social relations.

RMT assumes that the four scale types of Stevens (1946) are basic to social cognition. These four types are familiar from everyday life, illustrated in Table 1.

Scale types	**Examples**	**Examples**
Nominal	Pass versus fail	Hot versus cold
Ordinal	Student ranking	This is hotter than that
Interval	Letter grading	Centigrade
Ratio	Percentage grading	Kelvin

Table 1: Illustrations of Stevens' (1946) typology of scale types.

According to RMT, each basic mental model used in structuring social cognition strongly resembles one of the scale types. There are, thus, four basic forms of social interaction, one for each type of measurement scale.[7] Quoting the originator of RMT:

> Relational models theory is simple: People relate to each other in just four ways. Interaction can be structured with respect to (1) what people have in common, (2) ordered differences, (3) additive imbalances, or (4) ratios. When people focus on what they have in common, they are using a model we call Communal Sharing. When people construct some aspect of an interaction in terms of ordered differences, the model is Authority Ranking. When people attend to additive imbalances, they are framing the interaction in terms of the Equality Matching model. When they coordinate their actions according to proportions or rates, the model is Market Pricing. (Fiske 2004a: 3)

In Communal Sharing, people focus on what they have in common. This could be friendship, ethnicity, a history of suffering, romantic love, sharing food at the dinner table, etc. Fiske not only notes that nominal number assignments resemble Communal Sharing, he identifies Communal Sharing (CS) as a nominal scale:

> Roughly speaking, Communal Sharing is a kind of categorical (nominal) scaling, in that the only distinction that people make is of type or class: are two people of the same kind or different? Within the framework of the CS relationship, if two people belong to the same category (say, a family or ethnic group) then, with regard to the dimension that is communally organized, the people in that relationship are equivalent and undifferentiated. (Fiske 1991: 209)

An example of nominal scaling in measurement would be a questionnaire asking whether people smoke. Even though smoking is a matter of degree, this is ignored in nominal scaling. Either one belongs in the category of smoker or one does not. In Communal Sharing, likewise, one is either in the group or one is not. Shades of gray are not recognized.

Authority Ranking (AR) concerns ordered differences. Social units constitute a hierarchy. It resembles an ordinal scale, namely "a scale on which data are shown simply in accordance with some order, in the absence of appropriate units of measurement. For example, a squash ladder is an ordinal scale since one can say only that one competitor is better than another, but not by how much" (Borowski & Borwein 1991: 423). "Authority ranking is a linear ordering in which everyone's rank can be compared with everyone else's […]. Thus, the relations that are socially significant in an authority ranking relationship are similar to those that are specified by an ordinal scale" (Fiske 1992: 690).

In Equality Matching (EM), one maintains balance and corrects imbalances. Examples include the democratic principle of one vote per person, equal distribution of food in a school cafeteria, people taking turns, etc.

[7] There is much controlled evidence supporting the claim that the scale types play a fundamental role in social cognition. A bibliography listing much of this work can be found at www.sscnet.ucla.edu/anthro/faculty/fiske/RM_PDFs/RM_bibliography.htm.

> Equality Matching relationships resemble an interval scale in that people can not only specify who owes what to whom, but also how much they owe. In order to determine whether they are even, people match or balance what each person has given and/or received, and they can assess how great the imbalance is. In EM, order is represented by the fact that owing someone two big favors is a greater debt (a greater asymmetry) than owing the person one small favor. But unlike an ordinal scale, in EM people take implicit account of how much they have coming to them. (Fiske 1991: 209)

In other words, EM presupposes something like a centigrade scale in which differences between people are measured using equal units.

In Market Pricing (MP) involves ratios or proportions, e.g. a judge finding the right level of punishment to fit a crime. Determining the right price for a commodity is perhaps a more obvious example. "The structure of MP […] closely resembles a ratio scale of measurement. A symbolic system of propositions and abstract logical operations makes possible complex manipulations of ratios" (Fiske 2004b: 126).

There is consilience between the calculus approach to non-verbal mental representation and RMT. In both, elementary forms of representation are defined in terms of measurement scales. Furthermore, the elementary relational models, according to RMT, can be combined to form compound models (Bolender 2011, Fiske 2011). This resembles the performance of operations on elementary propositions to form molecular propositions, as discussed in Section 3. The fact that there is strong empirical corroboration for RMT (Haslam 2004b) makes the consilience even more persuasive. The consilience looks like some degree of corroboration for the psychological reality of Wittgenstein's approach. The relational models of RMT look like a sub-set of the collection of possible calculi. Furthermore, just as elementary models are defined in terms of the four scale types, so the various Wittgensteinian calculi are as well. Framing the calculus approach in terms of Stevens' (1946) typology of scale types, a typology crucial to RMT, one can say that RMT is a fragment of a more general Wittgensteinian conception of how logical spaces are represented. Spaces of social possibility form a subset of logical spaces as such, on this approach.

The consilience runs even deeper than this, also embracing work on animal cognition. There is much evidence that measurement scaling is the proper way of conceiving how nonhuman animals represent states of affairs. In 1990, C. R. Gallistel published an introduction to an anthology of papers devoted to mental representation in non-human animals (a special issue of the journal *Cognition*). The papers addressed representations of space, time, number, categorization of stimuli, and social relations among conspecifics. Reflecting upon the data presented in the volume, Gallistel concluded that we should speak of representations in animal cognition in the same sense that one speaks of representation in mathematics, namely as a functioning isomorphism. He further concluded that in the discussion of isomorphism, one discerns measurement theory:

> Those familiar with the theory of measurement, as developed initially by Stevens (1946) and more recently by Krantz, Lute, Suppes, and Tversky (1971), will recognize the parallel between this use of representation and its use in measurement theory, where the principal task is to establish the

> necessary and sufficient empirical conditions for the existence of an isomorphism between a to-be-measured psychological variable (e.g., loudness) and some or all of the number field. The isomorphism depends on finding a suitable measurement procedure (scale), which maps from the psychological variable to numerical representatives thereof, and on the existence of a formal correspondence between combinatorial operations on the psychological variables (as manifest in, for example, "louder than" judgments) and numerical operations such as ">", "+", and "=". This correspondence permits one to draw valid inferences about the psychological variables from mathematical operations on their suitably determined numerical representatives. (Gallistel 1990: 1–2)

Later in his introduction, he also noted the relevance of an important advance in cognitive anthropology, namely Fiske's (1991) RMT (Gallistel 1990: 20). Specifically, he noted that the role of measurement scales in social cognition could be the tip of the iceberg, in that measurement scaling runs all the way down to the roots of mental representation.

There are deep similarities between Wittgenstein's Second Theory and RMT. Both view their subject matter as bundled into semi-independent sub-systems. Furthermore, these sub-systems are, in at least some cases, constructed. This is especially clear in RMT, in which cultural variation is a factor in determining which construction of scales is applied to a type of social relation. For Wittgenstein, these were calculi. In Fiske's relational models theory, sociology is understood in terms of semi-independent psychological models. Wittgenstein understood each system as a construction of measurement scales, in the limiting case just a single scale. The same can be said of the social relational models of Fiske.

For Wittgenstein, logical relations within the system are to be understood in terms of mathematical operations. The transitivity of the greater-than/less-than relation of ordinal scaling would be an illustration of such a logical relation. Recursion also plays an analogous role in Wittgenstein and in Fiske.

> Among the Moose, a polygamous "nuclear family", a *zaka*, lives together and cultivates together, sharing stocks of food and eating together on a daily basis. Such a group is part of a compound (also *zaka*) that pools labor and food intermittently, occupying an enclosed living unit composed of one or more polygamous families. These compounds are grouped in larger unnamed communal groups that share a common grinding platform (*neere*) and greet outsiders ("*yeela*") when they first enter the area each day. People in this CS group routinely help each other with housebuilding, floor and yard pounding, beer brewing, and the like. These units are grouped in a named lineage neighborhood (*saka*) that is a communal group for other purposes, including giving and receiving wives, and making collective sacrifices to the ancestors. A set of neighborhoods comprise [*sic*] a village, which makes collective sacrifices to the earth for community fertility, rain, and protection from epidemics. The village (or sometimes the neighborhood) often pools labor to dig water catchment basins or wells. A set of villages makes up a named section with a loose sense of identity, and a few sections comprise a chieftainship under a paramount chief (*kombere*). The chieftainships together make up the Moose region. (Fiske 1991: 151)

For Fiske, social relations, relative to a given model, are to be understood in terms of mathematical operations (Fiske 1991: Ch. 9). It is reasonable to hold that the range of social cognition described by Fiske illustrates the application of logic to the social domain. In other words, the logic of Wittgenstein and the theory of Fiske stand in a genus/species relationship.

There is an historical irony here. For Fiske, recognition of the fundamental role of measurement scales in sociality led him to conclude that sociality is fundamentally simple. For Wittgenstein, however, the attempt to ground logic in measurement scales ended in frustration. Wittgenstein did not perceive an underlying unity in the types of measurements scaling. Whereas earlier, in *Tractatus* (Wittgenstein 1922: §§ 5.4541 & 6), Wittgenstein thought he had revealed a single simple underlying logical form in the appeal to truth functions, in his new appeal to measurement scales he could only perceive unstructured diversity. This ultimately led him to reject any attempt at theory, to view logical form, and even language as such, as a matter of loose family resemblances with no underlying essence (Wittgenstein 1974: 112–114, 2001: §§65–71).

In 1991, Fiske was in a good position to recognize a strong degree of unity and coherence among the types of measurement scales because of pioneering work in the theory of measurement conducted from the late 1940s through the 1980s. By extension, he was in a position to recognize a strong degree of unity and coherence in social cognition. Wittgenstein, by contrast, was pursuing his new theory of logic circa 1929, working without the benefit of these major advances. When Stevens published his watershed paper "On the Theory of Scales of Measurement" (Stevens 1946), Wittgenstein had already largely completed *Philosophical Investigations*. Not only was his anti-theoretical stance firmly entrenched by that time, there is no evidence that Wittgenstein, who died in 1951, was ever acquainted with Stevens' work. Furthermore, measurement theory continued to progress dramatically after 1951.

6. Logical Pluralism

The calculi approach is pluralistic. This is the case with regard to sociality specifically (Fiske) and with regard to logic quite generally (Wittgenstein). One can switch from one calculus to another in an effort to better conceptualize a domain of phenomena. It is perhaps even more evident that one will switch from one calculus to another according to the domain in question. There is empirical motivation for wanting an approach to natural logic with this sort of pluralism built into it.

More specifically, there is controlled evidence that people apply different logics to different subject matters. They will also sometimes shift from one logic to another for the same subject matter in an attempt to find the most suitable logic (Stenning & van Lambalgen 2008). On Stenning & van Lambalgen's interpretation of the data, the mind/brain imposes logical form on a task or puzzle, the form dictating the truth-preserving forms of inference. That one can shift between a fuzzy logic and a Boolean logic, according to the aims of one's inquiry, is perhaps obvious even before considering controlled evidence. In contexts in which it is useful to use terms such as 'frequently', 'slightly', etc., a fuzzy logic will be used. In the context of physics, one will often use a logic which is not fuzzy. Philosophers who discuss the identity conditions of the Ship of Theseus are, at least usually, assuming some kind of fuzzy logic. But this would not be the case if one were doing physics. "As far as the physicist is concerned, if you take out a nail, it's a new ship" (Chomsky 2012: 125). Even within a single field of endeavor, it may be rational to switch between logics, e.g. using a fuzzy logic to gauge symptoms ("severe headache", "frequent coughing"), and then a non-fuzzy logic to prescribe medication ("10 mg").

On this approach, it does not make sense to speak of a logic as being refuted. The question should be, rather: to what domain, if any, does a given logic apply? It is the assumption that a given logic applies to a given subject matter that is refuted, not the logic. Hence, the remarks of Danto (1988) and Hardin (1988) to the effect that experimentally induced experiences of reddish-green refute the sort of color logic found in, say, Wittgenstein's (1977) *Remarks on Colour*,[8] miss an important point. Danto and Hardin were justified in saying that such discoveries challenge philosophical arrogance and apriorism, and they may also have been correct in implying that Wittgenstein (or David Pears, whom Danto mentions without giving a full citation) were guilty of these. But the further implication that the observations (in e.g. Wittgenstein 1977) are in error is to overlook the fact that Wittgenstein's efforts can be interpreted as an investigation of *one possible logic* for making sense of color experience, presumably the one most commonly used. If one, in fact, experiences reddish-green in a laboratory setting, one can switch to a different color logic in order to accommodate the experience. This would be analogous to switching from Boolean logic to fuzzy logic when one discovers that the phenomena one assumed to be sharply discrete are actually rather amorphous. It also draws attention to the need for a generative approach to logic in which calculi are constructed as needed, just as social-relational models are constructed as needed.

Is there a space of possible logics, a kind of menu, from which one can choose? Stenning & van Lambalgen hesitate to say that there is:

> The approach to logic which we would like to advocate views logics from the point of view of possible syntactic and semantic choices, or what we will call parameter settings. This metaphor should not be taken too literally: we do not claim that a logic can be seen as a point in a well-behaved many dimensional space. (Stenning & van Lambalgen 2008: 25)

However, attempting to define such an all-encompassing space is worthwhile. Even if there is not such a space for all logics, there may be a space for a considerable number of them. To the extent that various logics can be defined in terms of a single space, one has achieved a degree of theoretical unification. Wittgenstein's Second Theory, as I argue in the next section, has the potential to provide a generative grammar of calculi, at least with regard to pre-verbal logics.

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8 This is the sort of logic in which the mixing of complementary colors is impossible, contrary to the grammar of the calculus in other words.

7. Self-Organization and Generativity

The Second Theory, understood in terms of the four scale types, suggests clues as to the brain activity underlying the range of humanly accessible logical forms. The question is how the mind/brain explores such a space, or can construct logics defining such a space. The current proposal is that a calculus is composed by the mind/brain. Calculi are constructed so as to represent logics appropriate to various subject matters. (This does not cover all variations among logics, since it does not address operators; the paper is only addressing one type of variation —variation at the atomic level.) Given evidence that various logics serve as tools for different domains, if one logic does not function adequately, one will try another. That is, if one calculus does not prove useful, one avails oneself of another for the same subject matter. This is consistent with recent evidence that people have options among contrasting logical forms in performing reasoning tasks (Stenning & van Lambalgen 2008). The aim is to find a calculus with the same logical multiplicity as the relevant subject matter, like trying to find the right dimensionality for the area one wishes to map.

The calculi in question would be pre-linguistic, and it is thus not plausible to view them as the result of a Merge-like operation (Hauser et al. 2002, Berwick & Chomsky 2016). But this does not mean that they are not the result of a generative grammar, for there are generative grammars which are not Merge-like. There are, for example, regular grammars (Chomsky & Miller 1958). The grammar in question here would produce an unstructured set of scale tokens, i.e. it would essentially be a tokenization procedure. The result of the procedure would be something closely analogous to a numeration (Chomsky 1995), but there would be no reason to posit a Select procedure as each scale would be immediately assigned a semantic interpretation without any need for producing hierarchical structure. The semantic interpretation would be its mapping to the relevant dimension of the given space. The construction of such 'numerations' would involve processes closely analogous to finite-state automata (Uriagereka 2008), and hence would not contradict claims as to the human uniqueness of language.

But how would such a tokenization process work, physically speaking? I believe that we do have some clues as to what sort of neurological process underlies the construction of measurement scale tokens, and thus have clues as to how the brain constructs or defines logical forms. One begins to delve into these processes by understanding the role of admissible permutations in measurement theory. Such a discussion reveals formal properties of the scales which, in turn, suggests a role for self-organization in the neurological process of tokenization.

Each scale type is characterized by a set of admissible permutations (Narens 1981, 2002), i.e. transformations which preserve information (e.g., the transformation of Fahrenheit readings into Celsius readings, the transformation of prices as a result of inflation). An admissible transformation is a symmetry. Hence, each scale type is characterized in terms of a set of symmetries. The four types of scaling bear a striking resemblance to animal gaits (e.g., walk, trot, gallop) (Bolender 2010). Each scale type has a corresponding group of symmetries, just as each animal gait exhibits a corresponding group of spatiotemporal symmetries (Buono & Golubitsky 2001, Golubitsky & Stewart 2015). Given that much work has been done on the role of the central nervous system in gaits, specifically their symmetries, this provides a clue as to the neural origins of the scale types. In both cases, one finds a descending chain of subgroups. The symmetries of one gait will be a subset of the symmetries of a different gait; "transitions between gaits break symmetry" (Stewart & Golubitsky 1992: 202). Likewise, the symmetries of a ratio scale are a subset of the symmetries of the corresponding interval scale. The symmetries of the latter are a subset of the symmetries of the corresponding ordinal scale. Its symmetries, in turn, are a subset of those of the nominal scale (Stevens 1946). In other words, the permissible permutations of ratio scaling are a subset of the permissible permutations of interval scaling, etc.

In both cases, namely gaits and measurement scales, this resembles self-organization. An illustration would be the structure of a snowflake. The symmetries of the snowflake are a subset, specifically a subgroup, of the symmetries of the droplet of water from which it formed (Stewart & Golubitsky 1992). In self-organization, one finds a sequence of such symmetry breakdowns, i.e. a chain of descending subgroups. One finds such a sequence, for example, in the successive transitions from plasma to gas, from gas to liquid, and from liquid to solid. Spontaneous symmetry breaking is evidently the ultimate source of structure in the universe (Close 2001). Roughly speaking, things tend to crystallize.

There is, evidently, self-organization among neural firing patterns as well (Buzsáki 2006). Neural oscillations illustrate the point. According to the mathematical biologists Ian Stewart and Martin Golubitsky, oscillations, owing to entrainment, exhibit patterns which can exhibit symmetry breaking and symmetry restoration. The rhythm of firing in a neural network is a temporal symmetry. Through Hopf bifurcation, the firing can change from a relatively more symmetrical to a less symmetrical pattern:

> The simplest way to describe Hopf bifurcation is as the onset of a wobble. The idea is that the system is influenced by some external variable as well as undergoing its own internal dynamics. At first the system is in a steady state, and does nothing; but as the external variable changes, a very slight wobble develops, which then grows until it becomes pronounced.
>
> (Stewart & Golubitsky 1992: 66)

Being due to spontaneous symmetry breaking, the resulting oscillatory pattern need not be the same as the temporal symmetries of the input firings (Golubitsky & Stewart 2015). Hence, this is a source of variation internal to the system, but also distinct from genetic coding. The various possible gaits for a single species evidently illustrate this. A number of distinct gaits can be modeled in a single hypothetical neural network via spontaneous symmetry breakdowns (Golubitsky et al. 1998, Golubitsky & Stewart 2015). One can account for various gaits in an animal by positing a single central pattern generator of connected neurons rather than a different network for each gait. On this approach, gaits are not learned, but neither are they encoded in the genome. Spontaneous symmetry breaking provides the organism with a repertoire of possible gaits.

The analysis of gaits by means of Hopf bifucation began by noting the symmetry group for each gait, thus observing how they form a descending chain of subgroups. This corresponds to our current level of progress in understanding how the brain represents measurement scales. A descending subgroup chain defines human measurement capacities. A possible next step is to create a computer model of a pattern generator producing the specific symmetries of each scale type via self-organization. It is such neural circuitry which is postulated as providing the raw material for creating the various calculi, specifically tokens of the various types of scaling. One would have an account of the multiplicity of logics, in terms of symmetry breaking. One would also have an account of the underlying unity behind logics, or at least the preverbal ones: The breakdowns of symmetry occur in a single, closely-knit neural system.

Stenning & van Lambalgen (2008) review a number of controlled studies which, on their interpretation of the evidence, support the view that natural logic is task relative. That is, for example, one is presented with a domain of percepts, and the mind's first task is to arrive at a logic suitable for that domain. The point in adopting a logic is "to aid in 'going beyond the information given' when processing information" (Stenning & van Lambalgen 2008: 16). To take a familiar example, if one has adopted a logic for color phenomena in which red and green are mutually exclusive, from the presence of green throughout a space-time region one can infer the absence of any admixture of red in that region. If it is very hot throughout a region, one can immediately infer that it is not very cold anywhere within it. They argue that reasoning is not only a matter of reasoning with a logic but also a matter of reasoning to a logic appropriate for the domain in question.

One may wonder how one is supposed to understand reasoning to a logic without presupposing some logic or other. The question is somewhat vague, as the notion of logic itself is somewhat vague. I suggest that one can make some progress toward answering the question by having at hand a plausible conception of how a logic is constructed. Whether or not the method of construction is to count as a kind of meta-logic is a question which can be bracketed, as one attempts to gain a foothold in the enterprise of how logics are constructed. Perhaps once one has some sense of how logics are constructed, one will be in a better position to answer this question. Understanding how a logic is constructed requires an idea of the possible constituents of a logic, and how those constituents are combined to form a logic. Arriving at such an idea can be facilitated if one can take the range of possible logics and somehow resolve it into constituent sub-domains, and then proceed with the analysis of one of those sub-domains as a starting point.

Returning to the work of Stenning & van Lambalgen (2008), we begin to get some sense of how the pre-linguistic mental faculties could choose among logics so as to accommodate the range of possibilities for a given perceptual domain (this is to use the word 'choose' rather loosely). Specifically, there is a choice among different types of measurement scaling and among a different possible number of scales. The point would be to arrive at a calculus that is homomorphic to the space/domain in question; that is, which shares the same logical multiplicity. Even without appealing to the empirical work discussed and Stenning &

van Lambalgen, it is introspectively obvious that one could have to switch from one calculus to another in an attempt to find the right logical multiplicity for a perceptual domain. This is apparently the case given some perceptual conditions which, even though rather exotic, are nonetheless easily described.

Very early on in formulating his conception of calculi, Wittgenstein was acutely aware of difficulties in determining the number of measurement scales to be used in mirroring the logical form of a given space: "One's first thought is that it's incompatible for two colors to be in *one* place at the same time. The next is that two colors in one place simply combine to make another" (Wittgenstein 1975a: §76). For example, one's first thought is that it's incompatible for red and yellow to be in one place at the same time. The next is that red and yellow in one place simply combine to make orange. Given that one reading on a measurement scale logically excludes all other readings, red and yellow can only occupy the same space simultaneously if there are two scales in play: one measuring the amount of red, and another measuring the amount of yellow. Hence, the choice between logical rules in this sort of case is, at least partly, a choice regarding the number of scales to be used.

Remarkably, by the late 1930s, if not earlier, Wittgenstein firmly rejected the possibility that choosing among logics is an empirically sensitive matter:

> It may be said that we recognize orange as reddish yellow because orange paint comes from red and yellow paint. But mixing paints cannot in a sense show us that orange is reddish yellow. Why shouldn't there be a chemical reaction?
>
> You might say, "That is not what we mean by mixing. We mean you use a colour mixer top". But suppose that when you spun it with red and yellow discs, the velocity made it go black. Would you then be inclined to say that black is a blend of red and yellow?
>
> So we do not use experience as our criterion for orange being a blend of red and yellow. For even if the paints and the top gave black, we should not call black a mixture of red and yellow.
>
> (Wittgenstein 1975b: 233–234)

I suggest that we pause and reflect upon the difference between what would actually happen while observing a red/yellow spinner top versus the kind of spinner-top experience which Wittgenstein is asking us to imagine. In the case of an actual red/yellow spinner top, as the top gradually accelerates there would be a point at which one has an ambiguous sort of experience. It would be a point at which one would have some difficulty ascertaining the proper number of measurement scales to apply. For, at that moment, one could either interpret one's experience as being that of orange revealing itself as having red and yellow constituents, or one could interpret the experience as simply being of red and yellow in rapid alternation. It is at this moment that the mind struggles to find the right logic, specifically the right number of scales for the calculus. The logical question, at this point, reveals itself to be empirically sensitive. One looks for clues in the visual experience to find the right logical multiplicity for the calculus.

By contrast, the thought experiment which Wittgenstein describes does not involve this element of the experience forcing one to find or create the appropriate logic. There are two different ways of interpreting Wittgenstein's example.

One is that he is asking us to imagine the colors red and yellow of the top alternating faster and faster until at some point the top is suddenly perceived as being black. No transition. There is nothing, however, in this sort of experience that would challenge the observer to find the right number of scales for mirroring the logical multiplicity of the color black. The experience, rather, would simply be of two colors suddenly being replaced by a third color. The other possible interpretation of Wittgenstein's thought experiment is that as the top spins faster and faster, the alternating pattern of red and yellow shades into black. The experience, in that case, would be similar to the 'fade to black' effect in film editing. However, once again, there is nothing in the experience that would challenge the observer to find the right number of scales to reflect the logical multiplicity of the color black. By contrast, in the realistic case in which the ever more rapid alternation of red and yellow merges into orange, there is a point at which the mind is challenged to find the right multiplicity. This is because, the red–yellow alternation is not simply fading into orange. There is, rather, the curious effect of the red–yellow alternation reaching a point at which the perceiver begins to see the alternation as a kind of orange. The observer can either interpret what they see as rapid alternation of red and yellow, or as orange with its internal structure laid bare. There is an in-between point at which the observer has some trouble judging which logic to use, in other words how many dimensions the calculus should have. The observer is experiencing some pressure to recognize an extra dimension (scale) so as to accommodate the apparent structure of orange. In other words, there is a transitional point at which the observer actually has the experience of orange as being constituted by red and yellow. There is nothing like this for the transition which Wittgenstein described from yellow/red into black.

Furthermore, this transitional stage is one in which the experience itself could force the observer to add a scale to the relevant calculus, thus resulting in a distinct calculus. It shows an experience forcing a change in logic.

8. Summary

In early Intermediate Wittgenstein, we find the following conception of mental representation: There are calculi which function to represent and capture logical relations in relative independence of one another. Each calculus represents a logical space, a range of conceptual possibilities, partly by reason of being isomorphic with the pertinent space. Some of these logical spaces may be mental constructions, while others may not be. The outputs of calculi can be combined using a recursive operation (Wittgenstein 1922: §5.2521) which serves as a point of connection between the various calculi. In terms of cognitive science, Chomsky's (1995) Merge operation is one possibility. A given pre-linguistic calculus is constructed from measurement scales. Given later developments in measurement theory, we can say, in hindsight, that there is the sort of beauty in these calculi evidencing self-organization. Hence, some properties of the calculi may 'come for free' in a manner similar to how economy conditions in syntax 'come for free', if they are indeed the result of self-organization.

Aiming to use Wittgenstein's Second Theory to further develop an account of semantics within the biolinguistic framework suggests a number of more specific projects. One such project is to use advances in measurement theory to better understand the range of possible calculi. For example, the possible scales of measurement are not limited to the four types discussed in Stevens (1946). There is also discrete interval scaling (Narens & Luce 1986). But note that all the possible types of scaling belong to the descending subgroup chain discussed earlier. Discrete interval scaling, for example, is located between interval scaling and ratio scaling, in the subgroup chain. Mention of the descending subgroup chain brings us to another project, namely understanding how the mind/brain constructs calculi in terms of spontaneous symmetry breakdowns in neural activity. Much brain activity is evidently the result of self-organization (Buzsáki 2006), and one wants to understand how the role of measurement scales in the construction of calculi is the result of such self-organizing activity, and precisely which self-organizing activity in the brain is in question (Bolender 2010).

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